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Sorting into Contests: Evidence from Production Contracts

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Abstract:
In this paper, we investigate sorting patterns among chicken producers who are offered a menu of contracts to choose from. We show that the sorting equilibrium reveals a positive sorting where higher ability producers self-select themselves into contracts to grow larger chickens and lower ability types self-select themselves into contracts to grow smaller birds. We also show that eliciting this type of sorting behavior is profit maximizing for the principal. In the empirical part of the paper, we first estimate growers' abilities using a two-way fixed effects model and subsequently use these estimated abilities to estimate a random utility model of contract choice. Our empirical results are supportive of the developed theory.

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1 Introduction

Pay-for-performance ties an employee's pay to his/her performance on the job. The idea is that pay-for-performance compensation schemes not only offer incentives to motivate and reward improved performance but also attract and retain better employees. These two interrelated roles, incentive and hiring, become one of the main subjects of study among personnel economists. In a recent survey, Oyer and Schaefer (2011) pointed out that personnel economics has made more progress in the area of understanding of how incentives work than on the subject of matching employees and firms.[1] In particular, relatively little is known about the mechanisms through which firms strategically design compensation packages to hire and retain appropriate workers. A key obstacle to advancement in this area, as being claimed, has been the paucity of integrated evidence due to the lack of usable real markets data.

The first empirical work in this area is Lazear (2000) who, relying on the Safelite Glass Corp. data, investigated the effect of changing compensation schemes on the productivity of windshield installers. He found that worker's average productivity increased by 44% after switching from fixed salaries to piece rates and half of the resulting productivity increase was attributable to attracting and retaining more able workforce. Barro and Beaulieu (2003) studied the effects of transferring physicians from a salary based compensation to a profit-sharing system. They found that the change had a large and significant effect on the quantity of services provided. In addition, they also detected a sorting/selection effect, where the least productive doctors left the hospital and more productive doctors joined. More recently, Bandiera et al. (2015) used administrative sources and survey data to study the match between firms and managers who are different in risk-aversion and talent. They found that policies with tighter link between performance and reward attract managers who are more talented and less risk-averse and also that managers respond to incentives by exerting more effort if offered steeper contracts.

In this paper, we use a unique and detailed data set which documents the settlements of contracts for the production of broiler chickens between a company and its contract growers.[2] The common characteristic of virtually all modern broiler production contracts is that contract growers' compensation is determined in a cardinal tournament setting where the individual grower's piece rate compensation is determined by comparing her performance against the tournament group average performance (see Knoeber and Thurman 1994; Levy and Vukina 2004). In our data set, we observe heterogeneous ability growers who participate in five different broiler production contracts offered by different complexes of a large broiler company in one geographic area. All contracts are the same when it comes to division of responsibilities for providing inputs: growers provide housing facilities, utilities and labor and the company provides birds and feed. Contracts are different with

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respect to two principal features: the size of chickens that need to be grown and the payment schedule offered in return. All contracts are written on a take-it-or-leave-it basis and are explicitly short-term (flock-by-flock) so we can observe multiple contract realizations for each individual grower. The main objective of this paper is to investigate whether contract growers choose among available contract alternatives based on systematic self-selection/sorting mechanism that can be uncovered in the data.

In addition to the above-mentioned empirical literature dealing with selection into alternative remuneration schemes and organizations, our paper is also connected to the theoretical literature on selection into contests. For example, Leuven, Oosterbeek, and van der Klaauw (2011) investigated how heterogeneous agents choose among contests with different prizes. Their main finding is that perfect sorting (high-ability agents compete for high prize and low-ability agents for low prize) is not necessarily obtained. Mixed strategies and reverse sorting are also possible. Azmat and Möller (2009) studied how competing contests should be structured to maximize participation. Their model with identical abilities contestants predicts that an increase in sensitivity with which contest outcomes depend on efforts makes flatter prize structures more attractive. In equilibrium, contests that focus on maximizing the number of participants will award multiple prizes if and only if this sensitivity is sufficiently high. Azmat and Möller (2013), in a model with binary abilities, showed that the distribution of abilities plays a crucial role in determining the contest choice. Sorting exists only when the proportion of high-ability contestants is sufficiently small. Morgan, Morgan, Sisak, and Várdy (2014) studied how large, heterogeneous population of risk-neutral agents self-select across two mutually exclusive contests. They showed that entry into richer contests was non-monotone in ability. This seems to be the only paper which characterizes selection when contests differ in multiple dimensions (entry fees, number of prizes, value of prizes and discriminatoriness/meritocracy) simultaneously.

One of the most difficult obstacles to overcome in empirical studies with non-experimental data is that the choice of compensation schemes in a firm is correlated with observable and unobservable characteristics of the firm and can rarely be considered truly exogenous. The use of controllable laboratory experiments is in this respect attractive because the exogeneity of the change in the compensation scheme is guaranteed by design. Cadsby, Song, and Tapon (2007) and Eriksson and Villeval (2008) examined the differences between pay for performance versus fixed salary in experimental settings and found evidence of positive sorting reflected in more productive workers choosing performance pay over fixed salary. Similarly, Dohmen and Falk (2011) comparing output of workers in fixed and variable payment schemes (piece rate, tournament and revenue-sharing) found that variable payment schemes attract more capable workers. They also found that change in the compensation schemes has multidimensional sorting effect with respect to other workers' characteristics such as risk aversion, relative self-assessment and even gender. Leuven et al. (2011) analyzed the sorting effects within the framework of rank-order tournaments. In their experiment, introductory microeconomics students self-selected themselves to different tournaments with low, medium and high prizes. Their results showed that the positive relationship between student's productivity and prizes were entirely attributable to the sorting effect where participants with higher ability were more likely to sort themselves into a higher reward tournament.[3]

Outside the experimental literature, theories of self-selection and sorting into contests have been empirically tested mainly with the sports data.[4] For example, earlier mentioned Azmat and Möller (2009) found empirical support for their findings with professional road running data and Azmat and Möller (2013) used entry data into marathon races for testing their theory. Finally, Lynch and Zax (2000) also relied on professional road racing data and found that races with large prizes record faster times because they attract faster runners and not because they encourage all runners to run faster. To the best of our knowledge, ours is the first attempt to test self-selection into contests based on the real market transactions data.

We start the paper by presenting a theoretical model geared toward deriving testable predictions. We show the existence of sorting equilibrium where higher ability growers sort themselves into contracts with higher expected output and lower ability types sort themselves into contracts with lower expected output. We also show that, from the perspective of the principal (integrator), the strategy of offering a menu of contracts that elicits positive sorting generates larger profits than offering one uniform contract to all growers. In the empirical part of the paper, we first estimate growers' abilities in a two-way fixed effect model and subsequently use these estimated abilities to estimate a random utility model of contract choice. The signs of the estimated model coefficients offer a direct empirical test of the developed theory. We showed that higher ability chicken growers are more likely to self-select themselves into contracts to grow larger chickens and the opposite is true for chicken growers with lower abilities.

2 Theoretical Model

The presented theoretical model describes a contractual relationship between a single principal and a number of heterogeneous ability agents. The principal simultaneously offers a menu of production contracts to a group of agents and each agent needs to decide which, if any, among the available contracts to sign. The number of agents in each contract N_k is exogenously determined by the principal based on his production needs. Once enough agents sign their chosen contracts, the production begins.[5] The production process within each contract (production division) is organized by a random assignment of agents into smaller tournament group of size n_{kt}, such that $n_{kt} < N_k, \forall t$. The optimal decisions are characterized by sorting equilibrium where each agent first chooses a contract that gives him the highest expected utility among all available contracts and then exerts optimal effort subject to parameters of the chosen contract. The model is designed to mimic the contracting process prevalent in the poultry industry where companies (integrators) contract the production of live chickens (broilers) with independent farmers (growers).

2.1 Stylized Facts and Preliminary Assumptions

Since all contracts are the same when it comes to defining the exact responsibilities of the contracting parties and other legal provisions, we assume that each contract can be uniquely identified by the payment schedule. In all modern broiler production contracts, the total payment R_{ikt} to grower i who participate in contract k and tournament group t is calculated as a variable piece rate times the live pounds of chickens harvested from the farm. The variable piece rate consists of two parts – the base rate and the bonus/penalty rate. The base rate b_k is common to all growers in contract k and its magnitude depends on the size of the chickens grown under that contract. Growing heavier birds takes longer and therefore requires higher base payment rate. The bonus/penalty rate is determined as a percentage β_k of the difference between the average performance of the entire group of growers whose chickens are harvested in the same week (i. e. growers in the same tournament t) and the individual performance of grower i. Algebraically, the total payment is computed as follows:

$$R_{ikt} = \left[b_k + \beta_k \left(\frac{1}{n_{kt}} \sum_{j=1}^{n_{kt}} \frac{C_{jkt}}{Q_{jkt}} - \frac{C_{ikt}}{Q_{ikt}} \right) \right] Q_{ikt}. \tag{1}$$

As seen from eq. (1), the grower's performance is measured by the total settlement cost C_{ikt}, which is the sum of expenditures for all production inputs supplied by the integrator (such as chicks and feed) divided by the total pounds of produced live weight Q_{ikt}. The above described relative performance scheme, frequently referred to as a two-part piece rate tournament, is a double-margin contest in which growers compete in producing as much output as possible with as little inputs as possible (see Tsoulouhas and Vukina 1999). The exact modeling of this tournament game is rather difficult because grower's effort to some degree stochastically influences both feed utilization and final output.[6]

To simplify, we assume that effort only influences grower's performance as measured by the negative of the settlement cost per pound of output and the performance function takes the following linearly additive form:

$$q_{ikt} = -\frac{C_{ikt}}{Q_{ikt}} = e_{ikt} + a_i + u_{kt} + w_{ikt}. \tag{2}$$

Growers are assumed to be heterogeneous in their abilities a_i, they exert efforts e_{ikt} and their performance is influenced by the common shock u_{kt} and idiosyncratic shock w_{ikt}. Each grower only knows her own ability and believes the abilities of all other growers in the same tournament are randomly drawn from a contract specific distribution $G_k(\cdot)$. The two production shocks are independent and identically distributed with zero means and distributions $F_{u_k}(\cdot)$ and $F_{w_k}(\cdot)$.

Total output Q_{ikt} is assumed to be randomly drawn from distribution $H_k(\cdot)$. Random distribution of total output is a consequence of the assumption that the mortality is determined by sheer luck and the fact that the initial number of chicks placed are approximately the same across all growers in any given contract. The assumption about random mortality is quite reasonable because broiler mortality largely depends on the quality of baby chicks and not very much on grower's effort. The initial placement of baby chicks is determined by the surface area of the standardized size housing facility and the targeted weight of fully grown birds.

Growers are assumed to be risk-neutral and their utility functions are given by $U_{ikt} = R_{ikt} - c(e_{ikt})$, where the cost of effort is a strictly convex function of effort. In particular, we assume a quadratic cost of effort $c(e_{ikt}) = \frac{\gamma}{2} e_{ikt}^2$ with $\gamma > 0$ and constant for all growers.

When choosing a contract from the menu of available contracts, growers observe the base payment rate b_k and the bonus/penalty coefficient β_k. An interesting characteristics of all five contracts in our data set is that they all have the same bonus/penalty coefficient equal to 1. Effectively, this feature simplifies the contract choice decision from the need to simultaneously choose the base and the bonus coefficients to choosing only the base rate. Because of the one-to-one correspondence between the base rates and target weights of broilers, by choosing the base rate, growers *de facto* choose the type/size of chickens that they will grow.

Notice that the realized output, the exact number of players in tournament groups and total compensations are not known to growers at the time of signing the contract. Growers assume that the number of growers in each tournament group n_{kt} is *i.i.d.* with distribution $L_k(\cdot)$. Production is stochastic and depends on common shocks (e. g. temperature and humidity) and growers' idiosyncratic shocks. Because tournaments filter out common production shocks, the payments in any particular contest (tournament) will vary in line with the composition of the tournament groups.

2.2 Self-selection Mechanisms

A growers will sign the contract which provides him with the highest expected utility among all available contract alternatives. Regardless of whichever contract the grower chooses, he will always exert the optimal level of effort which is determined by maximizing his expected utility:

$$\begin{aligned}
&\max_{e_{ikt}} \mathbb{E}(R_{ikt} - c(e_{ikt}))\\
= \quad &\max_{e_{ikt}} \int \cdots \int [b_k + \beta_k \tfrac{n_{kt}-1}{n_{kt}}(e_{ikt} + a_i + u_{kt} + w_{ikt} - \tfrac{1}{n_{kt}-1}\sum_{\substack{j=1\\ j\neq i}}^{n_{kt}}(e_{jkt} + a_j + u_{kt} + w_{jkt}))]Q_{ikt}\\
&\prod_{\substack{j=1\\ j\neq i}}^{n_{kt}} \mathrm{d}G_k(a_j) \prod_{j=1}^{n_{kt}} \mathrm{d}F_{w_k}(w_{jkt}) \cdot \mathrm{d}F_{u_k}(u_{kt}) \cdot \mathrm{d}H_k(Q_{ikt}) \cdot \mathrm{d}L_k(n_{kt}) - \tfrac{1}{2}\gamma e_{ikt}^2.
\end{aligned} \tag{3}$$

The first-order condition of the above maximization problem gives the best-response function for grower i given choices made by all other players in the same tournament:

$$e_{ikt}^* = \frac{\beta_k}{\gamma}\mathbb{E}\left(\frac{n_{kt}-1}{n_{kt}}\right)\mathbb{E}(Q_{ikt}). \tag{4}$$

Because, from the perspective of an individual grower, the expected number of growers $\bar{\mu}_k = \mathbb{E}(\frac{n_{kt}-1}{n_{kt}})$ and expected output $\bar{Q}_k = \mathbb{E}(Q_{ikt})$ in any tournament within the chosen contract are the same, expression eq. (4) can be simplified to provide the closed-form solution for the optimal effort. Note that the subscript t is dropped from the equation as grower's optimal effort is independent of tournament group that she belongs to.

$$e_{ik}^* = \frac{\beta_k}{\gamma}\bar{\mu}_k\bar{Q}_k. \tag{5}$$

Several important results are worth emphasizing. First, the formula for optimal effort shows that heterogeneous ability growers participating in the same contract will exert the same level of optimal effort, that is $e_{ik}^* = e_{jk}^*, \forall i \neq j$. This results hinges on the additive functional form specification of the performance function where the marginal utility of effort is independent of ability. Second, optimal effort is positively influenced by the slope coefficient β (power of incentives) and the expected output $\bar{Q}$ and negatively influenced by the disutility of effort parameter γ. Finally, growers will exert higher levels of effort if they are competing against more competitors. This is because in a cardinal tournament, extra effort not only increases one's own performance but also increases the average performance of the entire group which is used as the benchmark for comparison purposes. In larger groups, the effect of own effort on the average performance is diluted more than in the smaller groups, hence the incentive to work hard is more pronounced.

Because, in equilibrium, growers in the same tournament exert the same level of effort, after inserting the optimal effort eq. (5) into the utility maximization problem eq. (3), we obtain a tractable representation of grower's expected utility from participating in contract k:

$$\mathbb{E}U_{ik} = [b_k + \beta_k\bar{\mu}_k(a_i - \bar{a}_k)]\bar{Q}_k - \frac{1}{2\gamma}(\beta_k\bar{\mu}_k\bar{Q}_k)^2. \tag{6}$$

where $\bar{a}_k$ represents the average ability in contract k. Among K alternatives, grower i would choose to participate in contract k if $\mathbb{E}U_{ik} \geq \mathbb{E}U_{il}, \forall l = 1, 2, \cdots, K; l \neq k$. We use expression eq. (6) to obtain several interesting comparative statics sorting results. First, it is easy to see that if it were possible to choose the base payment b_k

independently of targeted output level $\bar{Q}_k$, regardless of their ability, all utility maximization growers would pool themselves into the production contract with larger base payment because eq. (6) is strictly increasing in b_k. Second, growers' utilities also depend on the slope β_k. However, as mentioned earlier, all five contracts in the menu have the same bonus coefficients ($\beta_k = \beta, \forall k$), hence, there is no sorting based on the slope (intensity) of the scheme.

Next, assuming for the time being, that from the perspective of an individual grower, the contract parameters, average abilities, and expected size of the tournaments are *ex-ante* the same across all contracts, an individual grower with higher (lower) ability should be more inclined to sort himself into contracts with larger (smaller) expected output. Such a comparative static sorting result could be derived by computing the partial derivative of grower's expected utility with respect to expected output:

$$\frac{\partial \mathbb{E}U_{ik}}{\partial \bar{Q}_k} = b + \beta\bar{\mu}(a_i - \bar{a}) - \frac{1}{\gamma}\beta^2\bar{\mu}^2\bar{Q}_k. \tag{7}$$

The proof is based on realizing that there exists a threshold ability $a_0 = \bar{a} - \frac{b}{\beta\bar{\mu}} + \frac{\beta}{\gamma}\bar{\mu}\bar{Q}_k$ such that for growers with abilities $a_i > a_0$, the expected utilities are increasing with output at $\bar{Q}_k$, that is, $\frac{\partial \mathbb{E}U_{ik}}{\partial \bar{Q}_k} > 0$, which means that they will sort themselves into contracts with expected outputs greater than $\bar{Q}_k$. On the contrary, growers with abilities $a_i < a_0$ will sort themselves into contracts with expected outputs lower than $\bar{Q}_k$ because their expected utilities are decreasing with output at $\bar{Q}_k$.

The presented sorting result is illustrated in Figure 1 which graphs grower's expected utility against the expected output levels for three growers with heterogeneous abilities, $a_H > a_0 > a_L$. Grower's expected utility is a quadratic function in $\bar{Q}_k$ represented by an inverted U-shape parabola. The expected utility curve for a high ability grower always lies strictly above that for a low ability grower because at each output level expected utility is always larger for the high ability type. Now let us analyze the contract choice problems for growers a_H, a_0 and a_L when facing the contract with expected output $\bar{Q}_k$. We can see that expected utility reaches its maximum for grower a_0 since $\frac{\partial \mathbb{E}U_{ik}}{\partial \bar{Q}_k}\Big|_{a_i = a_0} = 0$ and, hence, this grower should choose the contract with expected output $\bar{Q}_k$. For high ability grower a_H, her expected utility is increasing with output at $\bar{Q}_k$ and choosing contracts with output larger than $\bar{Q}_k$ generates higher utility. Therefore, grower a_H should choose contract with expected output $\bar{Q}_H^*$ to attain the highest possible utility $\mathbb{E}U_H^*$. On the other hand, the expected utility is decreasing with output at $\bar{Q}_k$ for low ability grower a_L which means that, in order to achieve highest possible utility, she should choose contract with output lower than $\bar{Q}_k$, namely $\bar{Q}_L^*$.

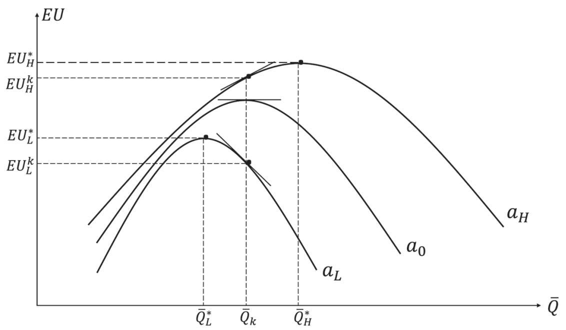

Figure 1: Sorting of heterogeneous ability growers based on expected output.

In fact, one can calculate the optimal expected output for each grower i by solving $\frac{\partial \mathbb{E}U_{ik}}{\partial \bar{Q}_k} = 0$ to obtain:

$$\bar{Q}_i^* = \frac{b + \beta\bar{\mu}(a_i - \bar{a})}{\frac{1}{\gamma}\beta^2\bar{\mu}^2}. \tag{8}$$

Expression (8) indicates that optimal output is an increasing function of grower's ability. If production contracts only vary by output, we expect to observe a positive sorting effect such that high ability growers self-select themselves into contracts with larger expected outputs while low ability growers choose to participate in contracts with lower expected outputs. The underlying rationale behind this result is straightforward. Higher ability growers have larger probability of winning the tournament competition. Therefore, choosing contracts with larger expected output can increase the potential bonus that they can earn. On the contrary, lower ability growers are more likely to lose the tournament and receive a penalty. Hence, it is wise for them to participate in contracts with smaller expected output to minimize the potential loss.

Similarly, assuming that contract parameters, average abilities and expected outputs are the same across contracts, it is easy to show that growers with higher (lower) abilities are more inclined to sort themselves into contracts with larger (smaller) number of players. This result can be explained by the influence of one player's performance on the tournament average. Higher ability players would prefer to compete with a large group of contestants because they don't want their good performances to significantly increase the average performance of the entire group which is used as a benchmark for comparison. Conversely, lower ability players prefer contracts with smaller number of contestants because, in this case, their poor performance can easily drag down the average performance making their penalty less severe.

2.3 Sorting Equilibrium

The self-selection results discussed so far are all based on the assumption that all contracts have the same exogenous average ability, not influenced by growers' optimal contract choices. However, the description of the self-selection equilibrium is more complicated because the equilibrium average abilities in available contracts are determined endogenously and vary with the change in contract attributes. Clearly, based on the previously obtained comparative static results, the average ability in contracts with larger output is, ceteris paribus, expected to be higher than that in contract with lower output. Hence, a rational high ability grower should anticipate that contracts with larger expected outputs should attract, beside herself, other high ability growers and, as a result, she may end up competing against a very strong pool of contestants. An alternative, perhaps a more profitable strategy, could be to sort herself into a contract with lower expected output but with potentially lower average ability pool of contestants which could guarantee an easy victory.

Therefore, as seen from eq. (6), the decision about which contract to choose involves a trade-off between the piece-rate determined by the grower's ability relative to the average ability in the respective tournament and the total expected output. Choosing a contract with large expected output would earn smaller expected piece-rate, whereas choosing a contract with smaller expected output would earn larger piece-rate. The optimal contract choice for each grower will depend on the multiplication of these two negatively correlated variables. The sorting equilibrium is summarized in the following proposition.

Proposition 1.
Suppose there are K contracts with increasing output levels $\bar{Q}_1 < \bar{Q}_2 < \cdots < \bar{Q}_K$, each with positive number of participating growers. Growers abilities follow the distribution $G(\cdot)$ with density $g(\cdot)$, minimum and maximum abilities are $a_{\min}$ and $a_{\max}$. There exist $K-1$ threshold abilities a_{12} < a *such that growers with abilities in $(a_{\min}, a_{12})$ choose contract with the smallest output $\bar{Q}_1$, growers with abilities in (a_{12}, a_{23}) choose contract with output $\bar{Q}_2$, and so on, and growers with abilities in $(a_{K-1,K}, a_{\max})$ choose contract with the largest output $\bar{Q}_K$. The contract average abilities are increasing in output levels, i. e. $\bar{a}_1 < \bar{a}_2 < \cdots < \bar{a}_K$.*

The proof of the Proposition 1 for the general case is relegated to the Appendix. Here we sketch the proof for the two contract case where $\bar{Q}_1 < \bar{Q}_2$. The established equilibrium is non-strategic in the sense that the individual grower's decision of which contract to enter cannot affect the average ability of a particular contract $\bar{a}_k$. This assumption hinges on the fact that the total number of growers in one contract is too large for the decision of one individual to have impact on the decisions of other individuals.[7] Based on eq. (6) and maintaining the assumption that $\bar{\mu}_1 = \bar{\mu}_2 = \bar{\mu}$, grower i's expected utilities from participating in contract $\bar{Q}_1$ and contract $\bar{Q}_2$ can be written as:

$$\mathbb{E}U_{i1} = [b_1 + \beta\bar{\mu}(a_i - \bar{a}_1)]\bar{Q}_1 - \tfrac{1}{2\gamma}(\beta\bar{\mu}\bar{Q}_1)^2$$
$$\mathbb{E}U_{i2} = [b_2 + \beta\bar{\mu}(a_i - \bar{a}_2)]\bar{Q}_2 - \tfrac{1}{2\gamma}(\beta\bar{\mu}\bar{Q}_2)^2.$$

Since both expected utility functions are linear in a_i, there exist a threshold ability a_{12}:

$$a_{12} = -\frac{b_2\bar{Q}_2 - b_1\bar{Q}_1}{\beta\bar{\mu}(\bar{Q}_2 - \bar{Q}_1)} + \frac{\bar{a}_2\bar{Q}_2 - \bar{a}_1\bar{Q}_1}{\bar{Q}_2 - \bar{Q}_1} + \frac{\beta\bar{\mu}(\bar{Q}_2 + \bar{Q}_1)}{2\gamma} \tag{9}$$

obtained by setting $\mathbb{E}U_{i1} = \mathbb{E}U_{i2}$ such that grower i with ability a_{12} is indifferent between choosing $\bar{Q}_1$ or $\bar{Q}_2$. The threshold ability a_{12} must be between $a_{\min}$ and $a_{\max}$ because both contracts have positive number of participating growers. It is easy to see that $\mathbb{E}U_{i1} > \mathbb{E}U_{i2}$ when $a_{\min} < a_i < a_{12}$ and $\mathbb{E}U_{i1} < \mathbb{E}U_{i2}$ when $a_{12} < a_i < a_{\max}$. Therefore, grower i will choose contract $\bar{Q}_1$ if her ability is between $a_{\min}$ and a_{12} and choose contract $\bar{Q}_2$ if her ability is between a_{12} and $a_{\max}$. Notice that the base payment b_k is not having any impact on growers' monotonic self-selection strategies. It only changes the values of the cut-off abilities. The two-contract sorting result is illustrated in Figure 2a.

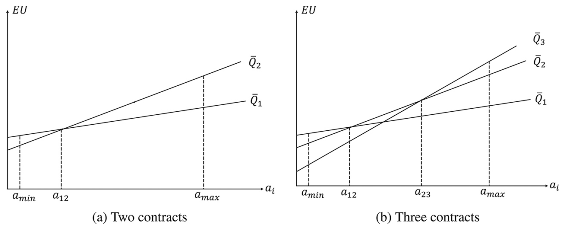

Figure 2: Non-strategic sorting equilibria.

The equilibrium average abilities are determined by the following two equations:

$$\bar{a}_1 = \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i}, \qquad \bar{a}_2 = \frac{\int_{a_{12}}^{a_{\max}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{\max}} g(a_i) \mathrm{d}a_i}. \tag{10}$$

Inserting a_{12} from eq. (9) as an upper limit of the integral in eq. (10) gives a system of two nonlinear equations with two unknowns $\bar{a}_1$ and $\bar{a}_2$. The equilibrium average ability in contract $\bar{Q}_1$ is smaller than in contract $\bar{Q}_2$ because $\bar{a}_1 < a_{12} < \bar{a}_2$, i. e.:

$$\bar{a}_1 = \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i} < \frac{\int_{a_{\min}}^{a_{12}} a_{12} g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i} = a_{12}$$
$$\bar{a}_2 = \frac{\int_{a_{12}}^{a_{\max}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{\max}} g(a_i) \mathrm{d}a_i} > \frac{\int_{a_{12}}^{a_{\max}} a_{12} g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{\max}} g(a_i) \mathrm{d}a_i} = a_{12}.$$

The result in Proposition 1 shows that the menu of contracts offered by the integrator company generates an equilibrium with monotonic positive self-selection of growers with heterogeneous abilities into contracts with differentiated expected output size. For high ability types the attractiveness of larger output outweighs the negative impact of higher average ability in the chosen tournament and potentially lower piece rate. Hence, it is optimal for these types to choose contracts with large output levels. On the other hand, for low ability growers the incentive to mix themselves with high ability types and enter the contract with large contracted output is too weak to overturn the chance of earning higher piece rate by competing against low average ability pool, leading them to self-select themselves into contracts with smaller expected output.

A parallel equilibrium where contract tournaments are separately varied by the number of growers can be easily obtained. In this equilibrium, high ability types would choose contracts with more players and low ability types would choose contracts with fewer players. The proof directly follows the proof of Proposition 1.[8]

2.4 Principal's Problem

Keeping the objective of hypotheses testing in mind, the usefulness of the result showing positive selection of growers with different abilities into contracts for growing different sizes of chickens hinges on the ability to prove that the whole scheme is meaningful from the integrator's perspective. This means that the principal offering a menu of contracts designed to motivate high ability growers to pick contracts for growing heavier birds and low ability types to pick contracts for growing smaller birds has to maximize her profit. There are two alternative possibilities that need to be evaluated.

First, instead of offering a menu of contracts which elicits positive sorting of growers into contracts, an integrator could instead offer a menu that would elicit negative sorting. This possibility is ruled out by the technological (nutritional) fact that feed conversion deteriorates with the size of animals grown. Because feed is the most significant production input, as a result, the cost of production per pound of live weight is always higher (worse) in contracts for heavier birds. Therefore placing better growers in contracts for larger broilers makes perfect sense because in this situation they can better manage feed and minimize the production cost.

Second, we need to show that offering a menu of contracts is better for the integrator (in the expected profit sense) than offering one uniform contract to all contract growers. Let us start by calculating the integrator's expected average profit from one grower in contract k as the difference between the expected average revenue and the average payment per grower and the average cost of production[9]:

$$\mathbb{E}\Pi_k = \mathbb{E}[p\bar{Q}_k - \frac{1}{n_{kt}} \sum_{i=1}^{n_{kt}} (b_k + \beta_k \frac{n_{kt}-1}{n_{kt}} (a_i - \bar{a}_k))\bar{Q}_k - \frac{1}{n_{kt}} \sum_{i=1}^{n_{kt}} C_{ikt}(e^*_{ikt})]. \tag{11}$$

Referring to the grower's performance function from eq. (2) and given that two production shocks are assumed to have zero means, expected cost of production becomes:

$$\mathbb{E}C_{ikt}(e^*_{ikt}) = -(e^*_{ikt} + a_i)\bar{Q}_k. \tag{12}$$

Substituting in the closed-form solution for optimal effort eq. (5), the integrator's expected profit per grower becomes:

$$\mathbb{E}\Pi_k = (p - b_k + \bar{a}_k)\bar{Q}_k + \frac{\beta_k \bar{\mu}_k \bar{Q}_k^2}{\gamma}. \tag{13}$$

To prove that a positive selection menu generates higher profits than a uniform contract we assume that the menu consists only of two contracts: contract H with larger expected output $\bar{Q}_H$ and contract L with smaller expected output $\bar{Q}_L$ and that higher ability grower would choose contract H and lower ability grower would choose contract L. The resulting expected profit of the principal under the positive sorting scenario is:

$$\mathbb{E}\Pi_k^P = (p - b + \bar{a}_H)\bar{Q}_H + \frac{\beta \bar{\mu}_H \bar{Q}_H^2}{\gamma} + (p - b + \bar{a}_L)\bar{Q}_L + \frac{\beta \bar{\mu}_L \bar{Q}_L^2}{\gamma}. \tag{14}$$

In the alternative scenario, the principal offers only one contract for both sizes of birds. We assume that half of growers will be tasked with the production of heavier birds (contract H) and the other half with the production of lighter birds (contract L). Since growers abilities are private information and there is no revelation mechanism in place, in expectation, the average ability in a uniform contract is $\frac{\bar{a}_H + \bar{a}_L}{2}$. The principal's expected profit from a uniform contract is:

$$\mathbb{E}\Pi_k^U = (p - b + \frac{\bar{a}_H + \bar{a}_L}{2})\bar{Q}_H + \frac{\beta \bar{\mu}_H \bar{Q}_H^2}{\gamma} + (p - b + \frac{\bar{a}_H + \bar{a}_L}{2})\bar{Q}_L + \frac{\beta \bar{\mu}_L \bar{Q}_L^2}{\gamma}. \tag{15}$$

The difference

$$\mathbb{E}\Pi_k^P - \mathbb{E}\Pi_k^U = \frac{(\bar{a}_H - \bar{a}_L)(\bar{Q}_H - \bar{Q}_L)}{2} \tag{16}$$

is strictly positive which proves the claim. The generalization of the result for the menu of three or more contracts is straightforward and is omitted for brevity.

3 Empirical Estimation

The objective of the empirical part of this paper is to test Proposition 1. For this purpose, we use contract settlement data from a large broiler company in the United States. The contract data set contains contract settlements information for five different broiler production contracts during a two-year period. These five contracts are differentiated by the size of the birds produced. The contracts for growing heavier birds have higher base payment rates. For the target weights varying between 4.8 and 6.2 pounds of live weight per bird, the base rate varies in the interval between 3.6 cents and 4.5 cents per pound. The slope parameter in all five contracts is equal to one. Table 1 presents summary statistics for several key variables. The data show that it usually takes 49–57 days for one-day old baby chicks to reach the target weight. For all five contracts, the feed conversion ratios are close to 2, which means that two pounds of feed is required for a bird to gain one pound of weight. The mortality rate fluctuates around 5% per flock per growing cycle.

Table 1: Summary statistics of broiler production contracts.

	Days		Target weight (Lbs.)		Feed conversion		Mortality rate	
Contract	**Mean**	**St. dev.**	**Mean**	**St. dev.**	**Mean**	**St. dev.**	**Mean (%)**	**St. dev. (%)**
A	49	1.48	4.94	0.25	2.08	0.06	5.23	2.58
B	50	1.63	4.81	0.29	2.03	0.08	2.90	3.31
C	56	1.98	5.86	0.25	2.19	0.12	4.66	2.96
D	57	1.55	6.01	0.20	2.17	0.06	4.43	1.73
E	57	1.86	6.21	0.27	2.19	0.11	5.43	3.14

Table 2 provides the summary of tournaments statistics. There are total of $K = 7{,}450$ observations and each observation provides one flock settlement information between the integrator company and an individual grower. The largest contract (production division) is B which has $N = 339$ growers under contract and the smallest is D

which has $N = 181$ growers. There is a tendency for the tournament groups (n) to be larger when the number of growers under contract is larger but the correlation is far from perfect. The production cycle takes 6-8 weeks to complete. Typically, tournament groups consist of all growers whose birds were harvested within the same calendar week, whereas the delivery of new batches is determined by scheduling and logistics of the production process.[10] The settlement costs are directly related to weight: the heavier the birds, the larger the costs of producing them.

Table 2: Summary statistics of contract tournaments.

				n in each T		Output (Lbs.)		Settlement cost ($)		Output per house per day (Lbs.)	
Contract	K	T	N	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.
A	896	48	199	19	3.16	232,233	119,821	75,658	39,326	1,987	597
B	3215	104	339	31	5.36	241,015	131,052	75,579	41,939	1,747	591
C	1388	104	307	13	3.22	334,021	139,281	106,666	45,289	2,294	611
D	947	76	181	12	2.65	304,768	170,074	101,469	56,548	1,910	648
E	1004	104	292	10	2.57	362,716	153,111	116,927	51,505	2,422	664

Note: K = number of observations; T = number of tournaments; N = number of growers in a contract; n = number of growers in a tournament.

The actual testing of the sorting result involves two steps. In the first step, we obtain growers' abilities by estimating a two-way fixed effect model based on the grower's additive performance function as specified in eq. (2):

$$q_{it} - e^*_{it} = a_N + \sum_{j=1}^{N-1} (a_j - a_N) d^j_{it} + \sum_{k=1}^{T-1} u_k g^k_{it} + w_{it} \tag{17}$$

where $q_{it} = -C_{it}/Q_{it}$ is the performance measure equal to the negative of the adjusted prime cost (APC) for grower i in tournament t and e^*_{it} is her optimal effort. As seen from eq. (5), optimal effort is not grower specific, and in the estimation procedure, optimal effort ends up being absorbed into the tournament fixed effects. APC measures the average cost accrued to the integrator of producing each pound of live broilers. It is computed as total settlement cost, which is the sum of cost of chicks, feed, fuel, medications, vaccinations and other customary flock costs charged to grower i, divided by the total pounds of live weight moved from the grower's farm.

We assume that ability a_j is a tournament-invariant variable specific to grower j[11] and common production shock u_k is a grower-invariant variable specific to tournament k. d^j_{it} and g^k_{it} are grower and tournament dummy variables with $d^j_{it} = 1$ if $j = i$ and 0 otherwise and $g^k_{it} = 1$ if k=t and 0 otherwise. To avoid singularity, only N-1 grower dummies and T-1 tournament dummies are included in the regression. That way, a_N is the ability of grower N and it is estimated as the constant term of the regression. Growers' fixed effects are the differences between each grower's ability and grower N's ability. Assuming the common shock in tournament T is zero, common shocks for all other tournaments are estimated as tournament specific fixed effects. Idiosyncratic shock of grower i in tournament t, w_{it}, is estimated as the error term.

In the second step, we test Proposition 1 by estimating a version of random utility model. The structure of our reduced form estimation equation is supposed to capture the essential features of both mentioned equilibria: where sorting is decided based on the size of birds grown and where sorting occurs based on the number of participating growers. We assume that before the first production contest (available in the data) is played, each grower was presented with a menu of five different contracts, each specifying which type/weight of broiler chickens will be grown and the parameters of the tournament payment scheme. With this information in hand, the grower needs to decide which contract to sign based on her own ability. Specifically, we assume that the utility that grower i expects to derive from participating in contract k is given by:

$$\mathbb{E}U_{ik} = \alpha_k + \lambda_Q (a_i - \bar{a}_k) \bar{Q}_{ik} + \lambda_N (a_i - \bar{a}_k) \bar{N}_{ik} + \epsilon_{ik} \tag{18}$$

where α_k is the contract-specific constant, a_i is the ability of grower i, $\bar{a}_k$ is the average ability in contract k, $\bar{Q}_{ik}$ is the expectation held by grower i about the daily output per chicken house in contract k and $\bar{N}_{ik}$ is the expectation held by grower i about the average number of growers per tournament. Growers' final total outputs largely depend on the target weight of finished birds (decided by the integrator) and the number of chicken houses they own. Since growing larger birds takes longer time to finish and operating more chicken houses means housing more birds, using pounds of live chickens per house per day is a measure of output which is comparable across growers and contracts, see Table 2 for comparison. The *iid* random error term ϵ_{ik} is assumed to follow a type I extreme value distribution which implies a multinomial logit model. Grower i would choose contract k if $\mathbb{E}U_{ik} \geq \mathbb{E}U_{il}$ for all $l \neq k$.

Since growers make their contract choices prior to the actual realization of their efforts and random shocks, the expected outputs and expected number of growers instead of the actual values are used to calculate grower's expected utility. In estimating eq. (18), $\bar{Q}_{ik}$ and $\bar{N}_{ik}$ are assumed to be exogenous and the same across all growers in a given contract. We calculate these two explanatory variables as the average output and the average number of growers in one tournament for the entire contract k.

Finally, notice the absence of the tournament slope parameter as a determinant of contract choice in the random utility model. This is because the slopes in all contracts are the same (equal to 1) so there is no choice to be made with respect to this contract attribute.

The expected signs of the estimated parameters are determined by the presented theory. If the estimated parameter λ_Q is positive, it means that growers with higher than average abilities would prefer contracts with larger expected output. The positive sign on λ_N would mean that higher ability growers sort themselves into tournaments with higher expected number of growers.

4 Empirical Results

Equation (17) was estimated by pooling together the data from all five contracts.[12] The parameter estimates are suppressed for brevity. Suffice it to say that high adjusted R^2 (around 0.9) indicates that growers' heterogeneous abilities and tournament common shocks capture most of the variations in growers' performances. There are two technical issues that we need to deal with during the estimation.

First, because the subsequent estimation of the logit models require growers' abilities which were not observed but rather estimated, the inference from models with generated regressors is invalid because the standard errors and test statistics are obtained without taking sampling variation into consideration (see e. g. Wooldridge 2002). To address this problem, we estimate both stages of the model using 1,000 bootstrap samples with replacement and the statistical inference relies on bootstrap standard errors. To account for clustering, the bootstrapping was performed by stacking observations that belong to the same agent into a vector. For example, if x_1, x_2 and x_3 belong to agent A, we treat $[x_1, x_2, x_3]$ as an observation; if y_1 and y_2 belong to agent B, we treat $[y_1, y_2]$ as another observation. Then we bootstrap these vectors with replacement. This method works in cases, such as ours, where there are many agents and the number of observations per agent is relatively small; for details see Cameron and Miller (2015).

Second, estimating fixed effects models could be hampered by possible incidental parameters problem; see Lancaster (2000). To deal with this issue, the bootstrapping procedure was implemented such that we use the entire data set (all growers) when drawing each of the bootstrap sample, but then after the sample has been drawn, we drop growers who participated in less than five tournaments and estimate the two-ways fixed effects and the multinomial logit models with this truncated sample.

The pairwise t-tests for the differences in estimated growers' abilities across contracts are presented in Table 3. The first column is the contract identifier, the second column presents the total number of growers, followed by the estimated mean ability and its standard deviation. In the next five columns, we show the pairwise differences in estimated abilities between the column contract and the row contract. The corresponding t-statistics are displayed in the parentheses beneath them. As we can see, the null hypotheses of equal average abilities cannot be rejected for only three contract pairs: *AC*, *AE* and *CE*. These results show *prima facie* evidence that heterogeneous growers are not randomly selected into contracts with different characteristics, but instead, there could be some systematic rules governing the selection process.

Table 3: Differences in estimated abilities across production contracts.

				***t*-test**				
contract	N_k	**mean(a)**	**std(a)**	**A**	**B**	**C**	**D**	**E**
A	149	0.0377	0.0058		0.0019*** (3.278)	-0.000174 (-0.270)	0.0058*** (9.680)	-0.000191 (-0.296)
B	321	0.0359	0.0057			-0.0020*** (-4.065)	0.0039*** (7.304)	-0.0021*** (-4.063)
C	281	0.0379	0.0066				0.0059*** (9.759)	-0.000017 (-0.031)

D	142	0.0320	0.0041	-0.0059*** (-9.792)
E	267	0.0379	0.0066	

Notes: Growers who participated in < 5 tournaments are dropped from the estimation. t-statistics are in parentheses; (*) indicates $p < 0.10$, (**) indicates $p < 0.05$, (***) indicates $p < 0.01$.

In addition, the results of the pairwise Kolmogorov–Smirnov test statistics shown in Table 4 reinforce the previous findings. The K–S statistics for a pair of contracts is calculated as $D_{ij} = \max_a(|F_i(a) - F_j(a)|)$, where $F_i(a)$ and $F_j(a)$ are the empirical cumulative distribution function of abilities of contract i and j. The results show that the null hypotheses that growers abilities of two production contracts come from the same (or identical) continuous distribution cannot be rejected for only three pairs of contracts: AC, AE and CE at 5% significance level. For the remaining seven pairs, the null hypotheses are rejected at 5% significance level (some are rejected at 1%) indicating a possible systematic sorting of heterogeneous abilities agents into contracts with different attributes. These results are also illustrated in Figure 3 which depicts the cumulative distributions of estimated growers abilities for all five contracts.

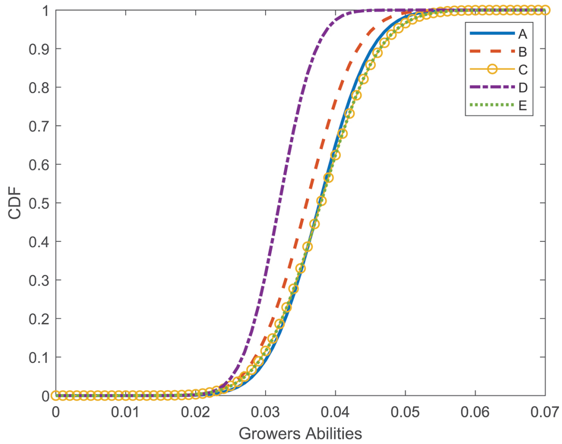

Figure 3: Distribution of growers abilities in broiler production contracts.

Table 4: Kolmogorov–Smirnov tests of differences in estimated abilities.

Contract	B	C	D	E
A	0.1573** (0.0114)	0.1115 (0.1665)	0.4595*** (3.51 E-14)	0.0943 (0.3464)
B		0.1933*** (2.16 E-05)	0.3753*** (8.58 E-13)	0.1762*** (1.94 E-04)
C			0.5255*** (1.15 E-23)	0.0185 (1.000)
D				0.5120*** (4.09 E-22)

Note: p-value are in parentheses; (*) indicates $p < 0.10$, (**) indicates $p < 0.05$, (***) indicates $p < 0.01$.

The estimation results of the logit model are summarized in Table 5. Several interesting results are worth highlighting. First, the contract-specific constant for contract D is assumed to be zero and the constants for all other four contracts are estimated in relationship to it. All estimated coefficients are positive indicating that relative to the left-out contract D, the remaining four contracts appear to be welfare superior to contract growers. However, none of those coefficients is statistically significant. Because, our theoretical model generates no predictions about the complete ordering of contracts, this result has no bearing on the empirical validity of our theory.

Table 5: Random utility model estimation results.

Parameter	Estimate	t-stat	p-value
α_A	4.9722	1.0871	0.2773

	(4.5739)		
α_B	5.0037	1.3537	0.1762
	(3.6965)		
α_C	7.7053	1.1039	0.2699
	(6.9804)		
α_E	7.4173	1.0572	0.2907
	(7.0159)		
λ_Q	0.3882	1.5680	0.1172
	(0.2476)		
λ_N	8.4745	1.6088	0.1080
	(5.2674)		
LL	-9986.9169		

Note: Standard errors are in parentheses.

Second, the estimated parameter of the greatest interest λ_Q has the theoretically correct (positive) sign indicating positive sorting of growers into contracts with different expected outputs based on their abilities. In other words, controlling for other unobservable contract attributes captured by the contract specific fixed effects $\alpha's$, higher than average ability growers are more likely to sort themselves into contracts with larger expected output (heavier birds) while lower than average ability growers would prefer contracts with smaller output levels (lighter birds). However, the parameter is only marginally significant with p-value of 11.7%.

Finally, the sorting behavior based on the expected number growers in one tournament, as captured by the estimated coefficient λ_N, is also shown to be positive. Again, controlling for other unobservable contract attributes, higher than average ability growers are more likely to sort themselves into contracts with larger tournaments (more players) while lower than average ability growers would prefer contracts with smaller tournaments (fewer contestants). Similarly, this parameter is also only marginally significant with p-value of 10.8%. Overall, the estimates of the random utility model largely confirm our theoretical predictions about mechanisms of contract choice.

5 Conclusion

Pay for performance compensation schemes have been used for quite some time in many sectors of the economy, such as manufacturing, agriculture and sales. Recently they have even penetrated several non-traditional sectors such as health care and education. Labor economics literature has recognized the fact that pay for performance can accomplish two tasks: it can provide incentives for workers to work hard and it can also help recruiting and retaining high ability employees who choose to work in highly competitive job environments. The literature on the provision of incentives has significantly outpaced the literature on hiring and job design. In particular, relatively little is known how firms design job packages to hire and keep workers they want. This paper contributes to the literature in this area in two important ways.

First, we develop a relatively simple theoretical model of sorting into cardinal tournament type of contests. The most interesting aspect of sorting into any type of relative compensation schemes is that high ability types face confusing and counter-balancing incentives. On one hand, they want to self-select themselves into a tournament with steeper incentives which they would surely do in any type of individual scheme such as simple piece rate. On the other hand, they would rather not select themselves into tournaments with steep incentives because they anticipate that other high ability types may also want to join the same tournaments which would make the competition in those tournaments rather stiff and the probability of winning rather remote. Instead, they might disguise themselves as low ability types and join a tournament with lower incentives but also with less severe competition which would earn them lower piece-rate but would surely improve their chances of winning the tournament. The incentives structure for low ability types is straightforward, i. e. they have no incentives to disguise themselves as high ability types and play in a tournament with high ability types. We were able to show that for high ability agents the first incentive trumps the second and one still obtains a separating equilibrium where high ability types choose tournaments with steeper incentives. To complete the model, we also showed that the chicken companies have the incentive to offer a menu of contracts which elicit positive sorting of heterogeneous ability contract growers into contract reflective of their types rather then offering *one size fits all* type of contract.

Secondly, we test the theoretical propositions with a unique real market data on the settlements of chicken production contracts. We were able to provide some evidence that, indeed, high ability types choose contract for growing heavier birds and low ability types choose contracts to grow lighter birds. This results makes perfect sense from the nutritional and genetic point of view. It is generally true that smaller animals have better (lower) feed conversion ratios than larger animals. So as chickens grow larger, their feed conversion ratio deteriorates. Because the integrator company always pays for feed and because feed is the single largest line item in the cost structure of live broilers production, the cost of production per pound of live weight is higher in contracts for heavier birds. Therefore, placing better growers in contracts for larger broilers makes sense because this job design places them into a situation where they can better utilize production inputs and minimize the production cost to the greatest extent.

Finally, our empirical results also show the tendency of high ability types sorting themselves into tournaments with larger number of players and low ability types into tournaments with fewer contestants, thereby affirming our theoretical prediction about positive assortative matching between abilities and the number of contestants in relative payment schemes (tournaments). This result is perfectly intuitive as well. High ability types prefer large cardinal tournaments where their good performance will only modestly improve the average group performance and raise the bar used for comparison. Conversely, low ability types prefer small tournament groups where their poor performance can substantially drag down the average performance, thereby lowering the bar for comparison and making their penalties less severe.

Of course, this work is not without some problems. Strictly speaking, the equilibrium selection of heterogeneous ability agents presented in the theoretical part of this paper pertains to contests differing in one dimension only (expected output or the number of players), yet in reality the selection occurs on two margins simultaneously. The theoretical characterization of sorting equilibrium for contracts with a relative performance evaluation function that differ in multiple dimensions simultaneously would be considerably more difficult to obtain. However, we speculate that this jointly determined equilibrium would most likely depend on the product of $\bar{\mu}_k$ and $\bar{Q}_k$ and since $\bar{\mu}_k$ is close to unity for many empirically observed tournament size, the equilibrium should decisively depend on the sorting based on the expected output and not so much on the number of players.

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Appendix

Proof of Proposition 1:

Proof. Suppose there are three contracts in the pool of contract alternatives with $\bar{Q}_1 < \bar{Q}_2 < \bar{Q}_3$. The threshold ability which equates the expected utility from contract $\bar{Q}_2$ and contract $\bar{Q}_3$,

$$a_{23} = -\frac{bN}{\beta(N-1)} + \frac{\bar{a}_3\bar{Q}_3 - \bar{a}_2\bar{Q}_2}{\bar{Q}_3 - \bar{Q}_2} + \frac{\beta(N-1)(\bar{Q}_3 + \bar{Q}_2)}{2\gamma N} \tag{19}$$

must be greater than a_{12} and smaller than $a_{\max}$. This is because when $a_{23} < a_{12}$, contract $\bar{Q}_2$ would be dominated by either contract $\bar{Q}_1$ or contract $\bar{Q}_3$ and would not be chosen by any grower, and when $a_{23} > a_{\max}$, even the best grower would pick contract $\bar{Q}_2$ and no grower would choose contract $\bar{Q}_3$. Hence, it must be that $a_{\min} < a_{12} < a_{23} < a_{\max}$. With this condition, growers with abilities in $(a_{\min}, a_{12})$ would choose contract $\bar{Q}_1$, growers with abilities in (a_{12}, a_{23}) would choose contract $\bar{Q}_2$ and growers with abilities in $(a_{23}, a_{\max})$ would choose contract $\bar{Q}_3$. This sorting result for three contracts is illustrated in Figure 2b in the main text. The equilibrium in average abilities is calculated as the solution of the following system of equations:

$$\bar{a}_1 = \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i)\mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i)\mathrm{d}a_i} \tag{20}$$

$$\bar{a}_2 = \frac{\int_{a_{12}}^{a_{23}} a_i g(a_i)\mathrm{d}a_i}{\int_{a_{12}}^{a_{23}} g(a_i)\mathrm{d}a_i} \tag{21}$$

$$\bar{a}_3 = \frac{\int_{a_{23}}^{a_{\max}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{23}}^{a_{\max}} g(a_i) \mathrm{d}a_i} \tag{22}$$

with eqs. (9) and (19) used as the thresholds (integral limits). Same as in the two contracts case, here as well, the equilibrium average abilities are positively related to the contract expected outputs because $\bar{a}_1 < a_{12} < \bar{a}_2 < a_{23} < \bar{a}_3$, i. e.:

$$\begin{aligned}
\bar{a}_1 &= \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i} < \frac{\int_{a_{\min}}^{a_{12}} a_{12} g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i} = a_{12} \\
\bar{a}_2 &= \frac{\int_{a_{12}}^{a_{23}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{23}} g(a_i) \mathrm{d}a_i} > \frac{\int_{a_{12}}^{a_{23}} a_{12} g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{23}} g(a_i) \mathrm{d}a_i} = a_{12} \\
\bar{a}_2 &= \frac{\int_{a_{12}}^{a_{23}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{23}} g(a_i) \mathrm{d}a_i} < \frac{\int_{a_{12}}^{a_{23}} a_{23} g(a_i) \mathrm{d}a_i}{\int_{a_{12}}^{a_{23}} g(a_i) \mathrm{d}a_i} = a_{23} \\
\bar{a}_3 &= \frac{\int_{a_{23}}^{a_{\max}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{23}}^{a_{\max}} g(a_i) \mathrm{d}a_i} > \frac{\int_{a_{23}}^{a_{\max}} a_{23} g(a_i) \mathrm{d}a_i}{\int_{a_{23}}^{a_{\max}} g(a_i) \mathrm{d}a_i} = a_{23}.
\end{aligned}$$

The same reasoning can be extended to K contract alternatives. Threshold abilities are expressed as

$$a_{k-1,k} = -\frac{bN}{\beta(N-1)} + \frac{\bar{a}_k \bar{Q}_k - \bar{a}_{k-1} \bar{Q}_{k-1}}{\bar{Q}_k - \bar{Q}_{k-1}} + \frac{\beta(N-1)(\bar{Q}_k + \bar{Q}_{k-1})}{2\gamma N} \quad \forall k = 2, 3, \cdots, K \tag{23}$$

and growers with abilities between $a_{k-1,k}$ and $a_{k,k+1}$ would choose contract k. The equilibrium average abilities can be calculated from the following system of equations:

$$\bar{a}_1 = \frac{\int_{a_{\min}}^{a_{12}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{\min}}^{a_{12}} g(a_i) \mathrm{d}a_i} \tag{24}$$

$$\cdots$$

$$\bar{a}_k = \frac{\int_{a_{k-1,k}}^{a_{k,k+1}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{k-1,k}}^{a_{k,k+1}} g(a_i) \mathrm{d}a_i} \tag{25}$$

$$\cdots$$

$$\bar{a}_K = \frac{\int_{a_{K-1,K}}^{a_{\max}} a_i g(a_i) \mathrm{d}a_i}{\int_{a_{K-1,K}}^{a_{\max}} g(a_i) \mathrm{d}a_i} \tag{26}$$

and, same as before, in equilibrium, contracted average abilities are increasing with contract expected output. □

Notes

1 Numerous empirical studies test for the existence of incentives effect and try to quantify its magnitude by examining the difference in performances under various compensation schemes; see Bull, Schotter, and Weigelt (1987), Ehrenberg and Bognanno (1990), Nagin et al. (2002), and Freeman and Kleiner (2005).
2 Poultry companies, frequently called integrators, such as Tyson Foods, Sanderson Farms or Perdue Farms almost never grow chickens on company-owned farms. Instead they contract the production of live birds with independent agents (farmers). Different profit centers (divisions or complexes) within a company typically specialize in production of a particular size/weight of birds and offer their own contracts to their growers.
3 For a recent comprehensive review of experimental research on contests and tournaments see Dechenaux, Kovenock, and Sheremeta (2015).
4 Tangentially related to our topic is also the empirical literature on the determinants of entrepreneurial selection, i.e. the propensity to become self-employed; see for example Van der Sluis, Van Praag, and Vijverberg (2008).
5 Typically, there is no shortage of potential growers for any contract. Poultry companies never complain about not being able to sign-up enough willing growers. To the extent that there is excess supply of willing prospective growers, the rationing takes place via first-come-first-sign mechanism until the target number is met.
6 A commonly used simplification is to fix one of the two margins. For example, Knoeber and Thurman (1994), Tsoulouhas and Vukina (1999), and Levy and Vukina (2004) fix the output margin by assuming common target weight of finished broilers and constant mortality rate. The alternative is to fix the cost margin and model the tournament as a contest of who can produce more output with given amount

of inputs, see Vukina and Zheng (2011). Neither of the two simplifications is suitable to our problem. This is because fixing one margin only works when modeling grower behavior in tournaments within one contract but not for the purpose of modeling how growers choose among different contracts.

7 Observe the magnitude of the N variable for various contracts in Table 2: 149 growers in contract A, 321 in B, 281 in C, 142 in D and 267 in E.

8 The equilibrium selection of heterogeneous ability agents into contests with vector-valued characteristics (expected output and the number of players) would be much more difficult to characterize; for an example see Morgan, Sisak and Vardy (2014).

9 Here we maintain the assumption that $\mathbb{E}(Q_{ikt}) = \bar{Q}_k$. The random distribution of total output is a consequence of the random mortality assumption and the fact that the initial number of chicks placed and the target weight of fully grown birds are the same for all growers. Therefore, for this particular technology and for a given contract, it is reasonable for the principal to believe that the total output is exogenous and that it is not correlated with growers' abilities.

10 The birds are harvested from a given farm when the integrator's production manager estimates that birds have reached the target weight and are ready to slaughter. The new cycle will start when the integrator delivers a new batch of birds to the farm. The acceptance of the new batch by the farmer constitutes a tacit renewal of the existing contract.

11 Traditionally, the literature defines agent's ability as innate characteristics or acquired skills based on education or experience. In this context, the definition of ability is extended to include any time-invariant grower idiosyncracies such as geographical location, the vintage and quality of housing facilities, etc.

12 One can imagine estimating a three-way fixed effects model with the third set of parameters being contract specific fixed effects. However, the structure of this data panel obviates the need for such a model because the tournament specific fixed effect capture the entire variation in contract types.

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