A Test for Market Power Using Marginal Input and Output Prices with Application to the U.S. Beef Processing Industry

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Diewert (1974, 1978) shows that standard duality models of profit maximization or cost minimization can be extended to account for the effect of imperfect competition by replacing average prices with marginal prices (i.e., marginal revenue and/or marginal factor costs of the firm). While this approach has been known and available for use in the literature for a number of years, it has only been applied in rather limited ways. In this paper we show how this method can be used to develop a general test for oligopoly and oligopsony behavior. The test exploits the fact that, under price-taking behavior, there is a known, fixed relationship between changes in input prices on output supply and changes in output price on input demands. The model and test developed are shown to be quite general and not dependent on empirical estimates of output demand and input supply.

We apply this test to the U.S. beef processing industry. Because of the high level of market concentration in this industry, there is concern that beef packing firms are exercising market power in the purchase of finished cattle by keeping cattle prices below competitive levels and in the sale of packed beef by keeping prices above competitive levels. Most previous studies of the beef packing industry have found evidence that firms, at least part of the time, are exercising market power in the purchase of finished cattle (Schroeter; Azzam and Pagoulatos; Schroeter and Azzam; Azzam; Azzam and Park; Koontz, Garcia, and Hudson) or are exercising market power in the sale of packed beef (Schroeter, Schroeter and Azzam). However, all of these studies are fairly restrictive in their assumptions regarding fixed proportions, the relationship between market power in the input and output markets, and the specification of input supply and output demand. Previous studies by Muth and by Muth and Wohlgenant (1999), which allow for variable proportions and do not impose restrictions on the relationship of market power in each market, did not find evidence of market power in the output and input markets for the beef packing industry; however, the results of each of these studies do depend on the specification of input supply and output demand.

Modeling the Effect of Imperfect Competition

Let p be the firm's output price, $q = f(\mathbf{x})$ the production function, $\mathbf{x}$ the vector of factor inputs, and $\mathbf{w}$ the vector of factor prices. Also, with exogenous demand and supply shifters subsumed in the functional notation, assume that $p = D(q)$ is the inverse demand function facing the firm for its output and $w_i = S_i(x_i)$ represents the supply function of the ith factor supplied to the firm. The central proposition of Diewert's (1974, 1978) work is that we can replace output price p by its marginal price $p^* = D(q^*) + q^*\partial D(q^*)/\partial q$ and input prices by their marginal prices $w_i^* = S_i(x_i^*) + x_i^*\partial S_i(x_i^*)/\partial x_i$ in the profit function, where q^* and x_i^* are the optimal output and input choices. Thus, we can write the profit function as

$$\max_{q,\mathbf{x}} [p^*q - (\mathbf{w}^*)'\mathbf{x}\colon q = f(\mathbf{x}),\ q \geq 0,\ \mathbf{x} \geq 0] = \pi(p^*, \mathbf{w}^*) = p^*q^* - (\mathbf{w}^*)'\mathbf{x}^*.$$

That is, maximum profit with marginal or shadow prices p^* and $\mathbf{w}^*$ substituted for average prices has the same value as the firm's

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true profit function. The reason this is true is because the modified profit function with marginal prices replacing average prices is a supporting hyperplane to the profit-maximization problem when the firm's output price is replaced by the demand function and when each factor price is replaced by its supply function (Diewert 1978).

In addition to the above specification, implementation of this model requires that the production function be concave (i.e., we have a convex technology). If $\pi(p^*, \mathbf{w}^*)$ is differentiable and q^* and $\mathbf{x}^*$ solve the above profit-maximization problem, then Hotelling's lemma implies

$$(1) \qquad q^* = \partial\pi(p^*, \mathbf{w}^*)/\partial p^*$$

$$(2) \qquad -\mathbf{x}^* = \nabla_{w^*}\pi(p^*, \mathbf{w}^*)$$

where $\nabla_{\mathbf{w}^*}\pi(\,)$ is the gradient vector of partial derivatives of the profit function with respect to $\mathbf{w}^*$, evaluated at $(p^*, \mathbf{w}^*)$.[1] If $\pi(p^*, \mathbf{w}^*)$ is twice-differentiable, then we can deduce the usual comparative statics results on the output supply relation and input demand relations.[2] If the production function is concave, then the partial derivatives of the output supply relation with respect to output price will be nonnegative and the partial derivatives of input demand relations will be nonpositive. There are also symmetrical, second-order cross-price derivatives of the profit function that imply symmetry restrictions between the output supply relation and each input demand relation, as well as between each set of input demand relations. These symmetry restrictions have the general form $\partial q^*/\partial w_i^* = -\partial x_i^*/\partial p^*$, $\forall i$ and $\partial x_i^*/\partial w_j^* = \partial x_j^*/\partial w_i^*$, $\forall i, j$.

To implement this method empirically, specifying the form of the marginal price variables is necessary because these are generally unobservable. Bresnahan (1989) (for oligopoly) and Muth and Wohlgenant (1998) (for oligopsony) show that, under fairly general conditions, the marginal output and input price variables at the firm level can be written as

$$(3) \qquad p^* = p(1 + \theta/\eta)$$

$$(4) \qquad w_i^* = w_i(1 + \phi_i/\epsilon_i)$$

where p and w_i are average market prices, η is the price elasticity of demand for the output, ϵ_i is the price elasticity of supply for the ith input, θ is the output conjectural elasticity, and ϕ_i is the input conjectural elasticity for factor i.

To obtain industry-wide expressions for equations (3) and (4), further assumptions are necessary for aggregation. In most studies of market power, one of two aggregation approaches is taken. The first approach is to assume that firms are identical in their technologies and thus have the same conjectural elasticities in equilibrium (see Schroeter). The second approach is to aggregate a firm-level expression by averaging it over firms or by summing over its share-weighted firm-specific components (see Kootnz, Garcia, and Hudson or Schroeter and Azzam). The approach taken in this study, which is outlined in an appendix available from the authors, falls into the latter category.

Diewert (1974, 1978) indicates that the marginal output and marginal input prices are unobservable and local information on the nature of the output demand and input supply functions is required. Equations (3) and (4) indicate that the nature of this information is summarized by the market demand and supply elasticities and conjectural elasticities. If one were interested in estimating specific values for these parameters, then it would be necessary to add specifications of output demand and input supplies to the set of equations to obtain estimates of these parameters. However, if one were simply interested in testing for the presence of imperfect competition, then equations (3) and (4) together with specifications for the elasticities in terms of exogenous variables would only be required. This is because, under price-taking behavior, $\partial q^*/\partial w_i = -\partial x_i^*/\partial p$, but when $p^* < p$ and $w_i^* > w_i$, $\partial q^*/\partial w_i \neq -\partial x_i^*/\partial p$. The question empirically then is how to parameterize η, ϵ_i, θ, and ϕ_i. In general, these are functions of all the exogenous parameters of the model because they depend on the values of the output (inputs) and determinants of demand (supply) at the point $(q^*, \mathbf{x}^*)$. With market data, this would imply all shifters of output demand and (endogenous) input supplies as well as determinants of the conjectural variation elasticities should be included. In the next section, we consider alternative parameterizations of these functions, including taking the elasticities as constant over the sample period. The specific empirical application is the U.S. beef

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[1] Expressions $\pi(p^*, w^*)$, q^*, and x^* are firm-level equations. Superscripts are omitted for convenience.

[2] Following Bresnahan's (1982) use of the term, "relation" describes the impact of changes in marginal prices on output quantities and input quantities.

processing industry where we allow for the presence of market power in the output market for beef and in the raw material market for cattle.

An Empirical Model of the Beef Packing Industry

The marginal prices in equations (3) and (4) derived above are inserted into a normalized quadratic profit function (see Diewert and Ostensoe) for a representative firm. The advantages of this functional form are that it is flexible and can allow for nonconstant returns-to-scale technology. By applying Hotelling's lemma to the indirect profit function and normalizing on an input used in beef production, w_n, we obtain expressions for output supply and input demand as follows:

$$(5)\quad q = \alpha_1 z_1\left[a_{22}\frac{p^*}{w_n} + a_{23}\frac{w_1^*}{w_n} + a_{24}\frac{w_2}{w_n} + \cdots a_{2n-1}\frac{w_{n-1}}{w_n}\right] + g_{20} + g_{21}z_1 + g_{22}\frac{1}{z_1}$$

and

$$(6)\quad x_1 = -\alpha_1 z_1\left[a_{32}\frac{p^*}{w_n} + a_{33}\frac{w_1^*}{w_n} + a_{34}\frac{w_2}{w_n} + \cdots a_{3n-1}\frac{w_{n-1}}{w_n}\right] + g_{30} + g_{31}z_1 + g_{32}\frac{1}{z_1}$$

where q is output of packed beef, p^* is the marginal output price of packed beef, x_1 is the input of finished cattle, w_1^* is the marginal input price for finished cattle, $w_2, \ldots, w_n$ are other input cost indexes or prices, and z_1 is a capital stock input. The last three terms in each equation allow for nonconstant returns to scale. Inputs other than live cattle are all assumed to be purchased in competitive markets.[3]

From the symmetry restriction on the profit function, the effect of the input price on output supply should be equal but opposite in sign to the effect of output price on input demand. Expanding the expression for the marginal prices in equations (5) and (6), the symmetry restriction implies that

$$(7)\quad a_{23}\left(1 + \frac{\phi_1}{\epsilon_1}\right) = -a_{32}\left(1 + \frac{\theta}{\eta}\right).$$

Thus, the estimated coefficients will be equal only if there is perfect competition in the input and output markets, that is, $\theta = \phi_1 = 0$.[4]

To implement this model, one must choose (*a*) the set of other input costs to include and (*b*) which input cost index or price to use as the deflator (w_n). Because the results may be sensitive to these choices, several variations were estimated and used for testing the symmetry restriction. Furthermore, as indicated above, one may estimate the coefficients on p and w_1 as constants or allow them to vary over time. A measure of concentration, such as the four-firm concentration ratio (CR4) and/or variables exogenous to the model are frequently chosen as variables in the coefficient expressions. Two alternative specifications were estimated here. In the first, a_{22}, a_{23}, a_{32}, and a_{33} were allowed to vary as functions of concentration and concentration-squared. In the second, a_{22} and a_{32} were allowed to vary with the price index for poultry and the CR4, and a_{23} and a_{33} were allowed to vary with the price of feed corn and the CR4.

Data Sources

The data used to estimate the preceding model are aggregate annual time-series data for the years 1966 through 1995. Data for the variables used in the model and for the instrumental variables were obtained from USDA's *Red Meats Yearbook*; USDA's *Livestock and Meat Statistics*; USDA's Animal Products Branch of the Economic Research Service; USDA's *Food Cost Review*; Bureau of Labor Statistics's *Employment, Hours, and Earnings*; the *Economic Report of the President*; USDA's *Food Consumption, Prices, and Expenditures*; and USDA's *Feed Situation and Outlook*.

[3] Input demand equations for the other inputs could have been derived as well, but the quantity data necessary to estimate these equations are not available and the test for imperfect competition does not depend on these equations.

[4] Note that θ/η cannot otherwise equal ϕ_1/ϵ_1 because θ and ϕ_1 are nonnegative, $\eta < 0$, and $\epsilon_1 > 0$.

Table 1. Comparison of Test Results for the Null Hypothesis of Perfect Competition in the Beef Packing Industry

Specification	Cost Indexes Included	Deflator	χ^2 Statistic	D.F.	p-value
Constant market power coefficients					
(a)	All	CPI	9.5262	1	0.0024
(b)	All	Transportation Index	10.2730	1	0.0017
(c)	All	Labor Index	10.3286	1	0.0017
(d)	All	Energy Index	11.0564	1	0.0014
(e)	All	Packaging Index	12.8888	1	0.0004
(f)	Marketing Cost Index	CPI	7.2045	1	0.0136
Coefficients as a function of poultry price, corn price, and concentration					
(g)	Marketing Cost Index	CPI	0.4712	3	0.9171
Coefficients as a function of concentration and concentration-squared					
(h)	Marketing Cost Index	CPI	3.1592	3	0.3750

Results of Estimation and Specification Testing

Equations (5) and (6) were estimated jointly by nonlinear three-stage least squares (3SLS) with first-order autoregressive error terms (quasi-first differencing). The parameter α_1 was set equal to one because it is not identifiable in these equations from the other parameters $a_{22}, \ldots, a_{2n}$ and $a_{32}, \ldots, a_{3n}$. Because data are not available on the quantity of capital stock used in the beef packing industry, trend was used as a proxy. Instrumental variables include the exogenous variables (marketing cost indexes and trend) as well as variables associated with wholesale demand for packed beef (consumption expenditures, population, pork prices, and poultry prices) and variables associated with the supply of finished cattle (corn prices and cattle inventories).

For each specification of the model, both the restricted ($a_{23} = -a_{32}$) and unrestricted specifications were estimated, and Gallant and Jorgenson's method of testing restrictions in nonlinear models was used to test the symmetry restriction. In addition, for each specification, Ljung-Box statistics were calculated at 6 and 12 lags to test the null hypothesis that the residuals are white noise. In each case, the null hypothesis was not rejected, thus indicating that the specification of the error structure is correct.

Table 1 lists the results of the restriction test for each specification. In all cases in which the market power terms were assumed constant [specifications (*a*) through (*f*)], regardless of the choice of deflator, the restriction was strongly rejected with p-values of 0.01 or less, thus implying the existence of imperfect competition in at least one market. This result held whether a complete set of cost indexes (transportation index, labor index, energy index, and packaging index) was included or a single overall marketing cost index was used. However, when the model coefficients were allowed to vary over time and the restriction was imposed locally at the sample means [specifications (*g*) and (*h*)], the restriction was not rejected, thus implying perfect competition in both markets. Thus, the conclusions regarding market power are very sensitive to whether one estimates the coefficients as constant or variable.

By making assumptions about the value of the output demand and input supply elasticities and assuming constant coefficients, one can also test the null hypothesis of monopoly and monopsony in each market. In equation (7), by letting $\theta = \phi_1 = 1$, $\eta = -0.53$, and $\epsilon_1 = 1.69$ (both of the latter values from Schroeter), a monopoly/monopsony restriction is obtained. When the restriction was imposed in specifications (*e*) and (*f*), it was strongly rejected (p-value $= 0.02$) in specification (*e*) but not rejected (p-value $= 0.22$) in specification (*f*). Thus, the results are inconclusive but again indicate the sensitivity of the results to the choice of specification.

Output supply and input demand elasticities were calculated for each of specifications (*g*) and (*h*). When evaluated at the sample means, the input demand elasticities for both the restricted and unrestricted models were negative, as expected, but had large variances. The output supply elasticities were negative at the sample means for both the restricted and unrestricted models, contrary to theory, but were positive at the upper end of the 95% confidence intervals. However, output supply elasticities could potentially be negative under imperfect competition if θ exceeds $|\eta|$ in equation (7). Finally, the results regarding returns to scale were tested in specifications (*g*) and (*h*) as well. The Gallant and Jorgenson test was applied to test the restriction that these coefficients intended to capture the effects of nonconstant returns to scale (g_{20}, g_{21}, g_{22}, g_{30}, g_{31}, g_{32}) are jointly equal to zero. The null hypothesis was strongly rejected (p-values $<$ 0.03) for both specifications regardless of whether the symmetry restriction was imposed or not. However, this is an aggregate model and these results must be evaluated in that context. Although constant returns to scale for the industry as a whole is rejected, it could be rejected in part due to the effects of the size distribution of firms. Individual firms may experience constant returns to scale, but when firms are aggregated, some with high-average costs and others with low-average costs, nonconstant returns to scale may appear at the industry level. However, it is important to include these effects in a test for market power.

Conclusions and Extensions

This model is more general than previous studies of market power in beef packing for four reasons. First, the model does not assume fixed proportions. Second, it allows the degree of market power in the input and the output market to differ. Third, the results of model estimation do not depend on empirical estimates of the input supply elasticity or the output demand elasticity. Finally, the model allows for nonconstant returns-to-scale technology. The results indicate that conclusions regarding imperfect competition are sensitive to whether the coefficients are estimated as constants or as functions of exogenous variables. In addition, for this particular application, it appears that returns to scale are nonconstant at the aggregate industry level.

One interpretation of observed negatively sloped supply relations is that the production function is not concave. An extension of the present approach would be to replace the profit function with a cost function and derive comparative static results with respect to changes in marginal input prices. By adding the relationship between marginal revenue (equivalently, marginal output price) and marginal cost to the set of factor demand relations, one would obtain a set of equations that could be estimated allowing for economies of scale in production. Although this may be a useful area for research, it is similar to the present approach except that the output price variable would be chosen as the variable to normalize the supply relation and the output quantity would be selected as an explanatory variable rather than as an output price on the right-hand side of the factor demand relations. Regardless of which approach is taken, the results suggest that the method for testing for market power is quite flexible and can be applied empirically with considerable ease.

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