Product heterogeneity and the relationship between retail and farm prices

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Summary
This paper examines the relationship between the price of an aggregate of agricultural outputs and raw material price in two situations: (i) when product heterogeneity within the product group arises from aggregation over heterogeneous commodities of different competitive industries, and (ii) when product heterogeneity arises from product differentiation among similar products in a monopolistically competitive industry. We show that a positive relationship between the aggregate price spread and the agricultural raw material price could result from input substitution between the raw material input and other inputs in response to changes in the relative raw material price. Indeed, within a composite product group there is likely to be significant input substitution, in response to changes in relative input prices, because of the increased opportunities for efficiency gains from altering the composition of the heterogeneous commodities within the composite product. Therefore, when analysing aggregate price spread behaviour of agricultural commodities using data on composite products, one should be cautious in attributing observed markup pricing behaviour to market power resulting from imperfect competition.

Keywords: aggregation, food processing, marketing margins, raw materials, variable proportions

JEL Classification: L11, Q11

1. Product heterogeneity and the relationship between retail and farm prices
The agricultural economics literature is replete with empirical studies on marketing margins for agricultural commodities, yet few of the studies provide theoretical underpinnings for the models estimated or rigorous explanations for the empirical findings. It is commonly asserted, for example, that marketing margins are determined as a markup of retail price over raw material, processing, and other marketing costs (e.g. Waugh, 1964; George and King, 1971), although the cause of such markups is unclear. One empirical regularity markup pricing is used to explain is the negative relationship between farm–retail price spreads and quantity of the raw material processed. In other words, when demand for an agricultural commodity at retail level is compared with demand for the raw material at the farm level, we often observe that...
demand at retail has a steeper slope than demand at the farm level. This indicates that the price spread narrows as quantity increases or, equivalently, that the price spread narrows as retail or farm price declines. Although markup pricing can account for such observed behaviour, such behaviour also can be explained by existence of significant input substitution between the raw material and other marketing inputs as well as economies of scale in food processing (Wohlgenant, 1989; Wohlgenant and Haidacher, 1989). Wohlgenant (1989) and Goodwin and Brester (1995) have shown that input substitution is significant for both aggregated and disaggregated US food industries; likewise, Morrison (1997) has demonstrated that economies of scale are also of some importance in accounting for markup pricing in US food industries.

The purpose of this paper is to evaluate more closely some of the significant causes of markup pricing in food processing industries. In particular, this paper extends the theoretical framework of Wohlgenant (1989) and Holloway (1991) by focusing on the effect of product heterogeneity on the relationship between retail and farm prices. Product heterogeneity can arise from two disparate sources: from aggregation over heterogeneous commodities of competitive industries, or from product differentiation among similar products in a monopolistically competitive industry. There are many ways to characterise aggregation over microeconomic units. The approach I take is somewhat limited in that only exact linear aggregation over individual firms is considered. The purpose of the present paper is to identify and highlight significant explanations for markup pricing and implications for farm–retail price spreads, not to develop a comprehensive approach to econometric analysis of retail and farm price relationships.

In the next section, the issue of heterogeneity in competitive industries, and its implications for retail to farm price relationships, is evaluated more closely. The third section extends the analysis to include the effects of product differentiation arising from monopolistic competition. The fourth section offers some concluding remarks.

2. Product heterogeneity and competitive industries

Let us first consider the simplest case where each firm in a given industry is a price taker and possesses a neoclassical production function with strictly diminishing marginal products. Then, for given output price $p$ and factor prices $w$, the firm’s cost function is increasing in output $y$ so that firm size is determinate. Now if there is also free entry into the industry and if minimum average cost firm output $y^*$ is small relative to total industry output $y$, then, as proved by Diewert (1981), the industry total cost function will be defined approximately as $yc(w)$ and industry output will be sold at a price $p = c(w)$, where $c(\cdot)$ is the unit cost function. Intuitively, the industry total cost function has this form when it is possible to replicate what existing firms are doing with no barriers to entry. One advantage of this specification of industry behaviour is that it is consistent with linear aggregation conditions under certain restrictions on output demand.
For the $i$th product, industry equilibrium can be characterised as

$$y_i^j = D_i^j(p, z)$$  \hspace{1cm} (1)$$

$$p_i^j = c_i^j(w)$$  \hspace{1cm} (2)$$

$$x_k = c_k^j(w)y_i^j \text{ for } k = 1, \ldots, m$$  \hspace{1cm} (3)$$

where $D_i^j()$ is output demand for the $i$th product, $p$ is the $n \times 1$ vector of commodity prices associated with the composite aggregate, $z$ is a vector of exogenous determinants of product demand (assumed to be the same for each product within the composite aggregate), $x_k$ is demand for the $k$th factor by the $i$th industry, and $c_k^j()$ denotes the partial derivative of the cost function of the $i$th industry with respect to the $k$th factor price. Equation (3) is obtained by Shephard’s lemma. Equations (1)–(3) characterise long-run equilibrium in the $i$th industry for given factor prices.

Let us assume that there exists a composite aggregate of the $n$ goods with price index $P = P(p)$ and quantity index $Y = Y(y)$, where $y$ is the $n \times 1$ vector of quantities of output $y_i^j$, $i = 1, \ldots, n$. These indices will exist, such that the product of the price and quantity index equals $PY$, if and only if each individual’s utility function is weakly separable in $y_1^j, \ldots, y^n$ such that the sub-utility function containing $y_1^j, \ldots, y^n$ is homogeneous of degree one (Green, 1964: Chapter 4). Moreover, individual consumers engage in two-stage maximisation wherein the second-stage demand functions can be written as

$$y_i^j = g_i^j(p)PY$$

for $i = 1, \ldots, n$. As these conditional demand functions are ordinary Marshallian demand functions, $g_i^j()$ is homogeneous of degree minus one, and these conditional demand functions can be written as

$$y_i^j = g_i^j(p_1/P, \ldots, p^n/P)Y = h_i^j(p)Y$$  \hspace{1cm} (4)$$

for $i = 1, \ldots, n$, where $h_i^j()$ is homogeneous of degree zero. In addition, the first-stage demand functions have the form

$$Y = D(P, z).$$  \hspace{1cm} (5)$$

Using the results that $y_i^j = h_i^j(p)Y$ from (4) and $p_i^j = c_i^j(w)$ from (2), aggregation over individual prices in (2) and individual input demands in (3) yields

$$P = \sum p_i^jy_i^j / Y = \sum c_i^j(w)h_i^j(c_1^j(w), \ldots, c^n(w))$$  \hspace{1cm} (6)$$

$$x_k = \sum x_k^j = \sum c_k^j(w)h_i^j(c_1^j(w), \ldots, c^n(w))Y, \text{ for } k = 1, \ldots, m$$  \hspace{1cm} (7)$$

where use has been made of the fact that $PY = \sum p_i^jy_i^j$. It should be noted that equations (6) and (7) could be derived directly from the aggregate cost function

$$C(w, Y) = c(w)Y = \sum wx_i^j = \sum c_i^j(w)h_i^j(c_1^j(w), \ldots, c^n(w))Y$$  \hspace{1cm} (8)$$
where $x^i$ is the minimum cost vector of inputs for industry $i$ associated with input price vector $w$. This aggregate function is a cost function because it satisfies the conditions for consistent aggregation (Chambers and Pope, 1991). To see this, we first recall that the index of output can be written as $Y = \sum p^i y^i / P$. Next, as shown by Chambers and Pope (1991: 812), consistent aggregation in this case requires that

$$\frac{\partial Y}{\partial y^i} \cdot \frac{\partial^2 C}{\partial y^i} = \frac{\partial^2 C}{\partial y^j}.$$

From the definition of $Y$, $\partial Y / \partial y^i = p^i / P$ and $\partial Y / \partial y^j = p^j / P$; also, from equation (8), $\partial C / \partial y^i = c^i$ and $\partial C / \partial y^j = c^j$. But $p^i = c^i$ and $p^j = c^j$ from equation (5). Therefore, condition (9) is satisfied for the aggregate function (8) because

$$\frac{(p^i / P)}{c^i} = \frac{(p^j / P)}{c^j}.$$

Because $C(\cdot)$ has the interpretation as an aggregate cost function, this means that the conditions (6) and (7) can be replaced by the equations

$$P = c(w)$$

$$x_k = c_k(w) Y \quad \text{for } k = 1, \ldots, m$$

where $c_k(\cdot)$ is the partial derivative of $c(\cdot)$ with respect to the $k$th factor price.

Let the first input, $x_1$, be the agricultural raw material. In general terms, the price spread or marketing margin, $m$, between the composite output price and agricultural raw material price can be defined as

$$m = P - (x_1 / Y)w_1.$$

Substituting from (10) and (11) yields the identity

$$m(w) \equiv c(w) - c_1(w)w_1.$$

Differentiating (12) with respect to $w_1$ (assuming $w_2, \ldots, w_m$ are exogenous) yields

$$\frac{\partial m}{\partial w_1} = c_1 - c_1 - w_1 c_{11}$$

$$\frac{\partial m}{\partial w_1} = -w_1 c_{11}$$

where $c_1$ is the partial derivative of $c(\cdot)$ with respect to $w_1$ and $c_{11}$ is the partial derivative of $c_1(\cdot)$ with respect to $w_1$. The function $c_{11}$ can be defined in terms of the Allen elasticity of substitution, $\sigma_{11}$, as follows (see, e.g. Diewert, 1974a):

$$\sigma_{11} = c(w) c_{11} / (c_1 c_1).$$

Solving for $c_{11}$ and substituting into (13) yields

$$\frac{\partial m}{\partial w_1} = -s_1 \sigma_{11} x_1 / Y$$

where $c_1 = x_1 / Y$ from (11) and $s_1 \equiv w_1 x_1 / c(w) Y = w_1 x_1 / PY$ is the cost share of raw materials in total costs (which equals total revenue under pure competition). The Allen elasticity of substitution, $\sigma_{11}$, is non-positive. Therefore, the sign of the partial derivative in (14) is non-negative. That is, the price spread is positively related to the raw material price under pure competition if
and only if the agricultural raw material and other inputs can be combined in variable proportions in producing the composite output $Y$. In the two-input case (or equivalently, when all other inputs can be combined into a single composite commodity (Diewert, 1974b)), $s_1 \sigma_{11} = -(1 - s_1) \sigma$, where $\sigma$ is the elasticity of substitution between the agricultural raw material and the composite index of other inputs. That is, in this case, markup pricing is directly proportional to the degree of substitutability between the agricultural raw material and other factors.

The fact that $\sigma_{11}$ in (14) is the elasticity of substitution associated with the aggregate cost function $C(\cdot)$ has special significance for interpreting the relative importance of input substitutability in explaining changes in farm–retail price spreads for agricultural commodities. In particular, equation (7) indicates that there are two sources of substitutability captured by the cost function $C(\cdot)$: (i) substitutability arising from changes in input–output combinations within industry $i$, as reflected in changes in $c_i^k$; and (ii) substitutability arising from compositional changes in the composite output as reflected in changes in $h_i^l$. Indeed, as shown in the Appendix, the substitution elasticity $\sigma_{11}$ can be viewed as a weighted average of substitutability arising from both sources (a) and (b); i.e.

$$\sigma_{11} = \sum_i (x_i^j / x_i)(s_i^j / s_i) \sigma_{11}^j + \sum_i \sum_j (x_i^j / x_i)(s_i^j / s_i) \eta_{ij}$$

(15)

where $s_i^j = w_i x_i^j / PY$, $\sigma_{11}^j$ is the Allen own-elasticity of substitution for the raw product within industry $i$, and $\eta_{ij}$ is the elasticity of demand for product $i$ with respect to product price $j$ within the composite output $Y$. Hence, substitutability between inputs for the aggregate commodity can be viewed as a weighted average of substitutability in production and substitutability in consumption. This means that the relative valuation placed on the single commodities within the aggregate by consumers affects the relative efficiency of the agricultural raw material in producing the output. Therefore, for anything other than the most elementary commodities, input substitutability is best viewed as an economic entity whose magnitude depends upon the subjective valuation placed on the different commodities produced from the agricultural raw material. Buccola (1997) comes to a similar conclusion for a different model in which producers maximise profit by selecting the optimal set of product designs and optimal quantity. The significant point about (15) is that input substitutability at the industry level is not required in order to have substitutability for the composite output when the output is obtained as an aggregate of heterogeneous products. In other words, each industry can produce its output with a Leontief technology and yet input substitutability would be observed in the aggregate of individual products because of compositional changes in output from changes in relative prices among the individual products within the composite output.1

1 Diewert (1981) shows that an additional source of input substitutability can arise from changes in the distribution of industry output among individual firms as input prices change. Thus, even the assumption of each industry producing its output via a Leontief technology seems tenuous.
3. Product differentiation and imperfect competition

Let us now attempt to generalise the results from the previous section by relaxing the assumption that firms are price takers. In particular, the assumption is now made that firms produce differentiated products and engage in Bertrand price-setting. Without loss of generality, we assume also that each product is produced by a single firm. We let \( n \) denote the number of firms and let each firm be indexed by \( i \). Then the profit maximisation problem for the \( i \)th firm is to choose \( p^i \) so as to maximise

\[
\pi^i = (p^i - c^i(w))D^i(p^i, \ldots, p^{i-1}, p^{i+1}, \ldots, p^n) - f^i \tag{16}
\]

where \( f^i \) denotes fixed costs of the \( i \)th firm. Under certain (rather weak) restrictions on consumer preferences across commodities within the product group, Caplin and Nalebuff (1991) prove that there exists a pure-strategy Bertrand–Cournot equilibrium for this problem. The equilibrium price of the \( i \)th firm has the form

\[
p^i = (1 + \beta^i)c^i(w) \tag{17}
\]

where \( 1 + \beta^i \) is the \( i \)th firm’s markup over marginal costs. If the consumer’s preferences are constant elasticity of substitution (CES) and there are a large number of products, then \( \beta = \beta \) where \(-1/(1 - \beta)/\beta \) is the elasticity of demand facing the firm (Dixit and Stiglitz, 1977). In addition to (17), the \( i \)th firm also purchases factors whose input demands are determined by Shephard’s lemma:

\[
x^i_k = c^i_k(w)y^i \text{ for } k = 1, \ldots, m. \tag{18}
\]

It should be noted that these equations are equivalent to those of the competitive industry, equation (3).

The question now becomes: under what conditions can we consistently aggregate equations (17) and (18)? The answer is that consistent aggregation is possible only if each firm has the same markup. To see this, it should be noted that for the imperfectly competitive firm, equation (9) indicates that consistent aggregation requires as before that

\[
(p^i/P)/c^i = (p^i/P)/c^i.
\]

However, from (17), \( p^i/c^i = (1 + \beta^i) \) and \( p^i/c^i = (1 + \beta^i) \). Therefore, this condition will be met only if \( \beta^i = \beta^i \), or \( \beta^i = \beta \) for all \( i \). That is, for the consistency requirement for aggregation across individual products to hold, it is necessary to replace equation (17) by

\[
p^i = (1 + \beta)c^i(w). \tag{19}
\]

Using the result that \( y^i = h^i(p)Y \) from (4) and conditions (18) and (19), aggregation over firms yields

\[
P = \sum p^i y^i/Y = \sum (1 + \beta)c^i(w)h^i((1 + \beta)c^1(w), \ldots, (1 + \beta)c^n(w))
\]

\[
x_k = \sum x^i_k = \sum c^i_k(w)h^i((1 + \beta)c^1(w), \ldots, (1 + \beta)c^n(w))Y
\]
for \( k = 1, \ldots, m \). It should be noted that, because \( h^i(\cdot) \) is homogeneous of degree zero in prices, these aggregate relationships have the equivalent representation

\[
P = (1 + \beta)c^i(w)h^i(c^1(w), \ldots, c^n(w)) = (1 + \beta)c(w)
\]

\[
x_k = \sum c^i_k(w)h^i(c^1(w), \ldots, c^n(w))Y = c_k(w)Y \text{ for } k = 1, \ldots, m
\]

where the last equalities in (20) and (21) follow from the definition of \( C(\cdot) \) in (8).

To analyse the impact of imperfect competition with product heterogeneity on the relationship between the composite output and raw material price, we can proceed as before by substituting (20) and (21) into the definition of the marketing margin, \( m = P - (x_1/Y)w_1 \), to obtain

\[
m(w) \equiv (1 + \beta)c(w) - c_1(w)w_1.
\]

Differentiating (22) with respect to \( w_1 \) (assuming \( w_2, \ldots, w_m \) are exogenous) yields\(^2\)

\[
\frac{\partial m}{\partial w_1} = (1 + \beta)c_1 - c_1 - w_1c_{11}
\]

or

\[
\frac{\partial m}{\partial w_1} = \beta c_1 - w_1c_{11}
\]

or

\[
\frac{\partial m}{\partial w_1} = (\beta - s_1(1 + \beta)\sigma_{11})x_1/Y
\]

where as before we have used the definition \( \sigma_{11} = c(w)c_{11}/(c_1c_1) \) and the result that \( w_1x_1/PY = s_1(P/c(w)) = s_1(1 + \beta) \).

As under pure competition (equation (14)), equation (23) indicates that the price spread is positively related to the raw material price under imperfect competition. However, in contrast to pure competition, the price spread will still be positively related to the raw material price even when there is no substitutability between inputs.\(^3\) However, when the composite product is produced with variable proportions of the agricultural raw material and other inputs (the more likely case), both imperfect competition and input substitutability can account for markup pricing behaviour. Among other things, this suggests a bias in empirical estimates of the degree of market power estimated assuming fixed input proportions, a point originally made by Wohlgenant and Haidacher (1989) and subsequently noted by Holloway (1991).

To obtain some feel for the empirical significance of input substitutability arising from product heterogeneity, let us consider the US beef industry

\(^2\) Differentiation of this function is conducted assuming that \( \beta \) is independent of \( w_1 \). Making \( \beta \) a function of \( w_1 \) would only complicate the analysis and not alter the basic conclusions.

\(^3\) Again, this analysis assumes that \( \beta \) is independent of \( w_1 \) or that any indirect (negative) effect of a change in \( w_1 \) on \( \beta \) is not large enough to offset the direct effect indicated by equation (23) when \( \sigma_{11} \) is zero.
where beef consists of two products: ground beef and table cuts. Using data from Brester and Wohlgenant (1997), we assume that 
\[ x_1^1/y_1^1 = x_2^1/y_2^1 = x/Y = 1.5, s_1^1 = 0.65, s_2^1 = 0.27, \text{ and } s_1 = 0.36. \]
We assume also that \( \beta = 0.2 \) so that, with the CES utility functions (Dixit and Stiglitz, 1977),  
\[ \eta_{11}^1 = -(1 + \beta)(1 - p_1^1 y_1^1/PY)/\beta = (1.02)(1 - 0.23)/0.2 = -4.62, \eta_{12}^1 = 4.62, \]
\[ \eta_{21}^1 = (1 + \beta)(p_1^1 y_1^1/PY)/\beta = 1.38, \text{ and } \eta_{22}^1 = -1.38. \]
Therefore, assuming \( x_1^1/x_1 = 0.42 \) (Brester and Wohlgenant, 1997) and assuming for the sake of argument that \( \sigma_{11}^1 = 0 \), we have, using equation (15), that  
\[ \sigma_{11}^1 = (0.42)((0.65/0.36)(-4.62) + (0.27/0.36)(4.62)) \]
\[ + (0.58)((0.65/0.36)(1.38) + (0.27/0.36)(-1.38)) \]
\[ = -1.20. \]
For the two-input case, this would imply an elasticity of substitution of 0.68 \( (\sigma = -(s_1/(1 - s_1))\sigma_{11}^1) \), which is strikingly close to the estimate of 0.72 by Wohlgenant (1989).

4. Concluding remarks

The principal finding of this paper is that, when analysing margin behaviour with composite products, one should be cautious in attributing observed markup pricing behaviour to market power from imperfect competition. A positive relationship between price spreads and the agricultural raw material price could just as well result from input substitution that occurs as the relative raw material price changes. Analysis of demand behaviour using composite products suggests that significant input substitution will be more prevalent because of the increased opportunities for efficiency gains from altering the composition of the heterogeneous commodities within the composite product.

References


**Appendix**

To derive equation (15), first we partially differentiate $x_1$ in equation (7) with respect to $w_1$ to obtain

$$\frac{\partial x_1}{\partial w_1} = \sum_i (\partial c_i^1/\partial w_1)h_i(\cdot)Y + \sum_i \sum_j (\partial h_i/\partial p_j)(\partial c_i^1/\partial w_1)c_i^1(\cdot)Y.$$ 

From the definition of partial elasticity of substitution (Diewert, 1974a),

$$\frac{\partial c_i^1}{\partial w_1} = s_{i1}(c_i^1c_j^1)/c_i^1(w).$$

Substituting into the above expression and multiplying both sides by $(w_1/x_1)$ gives

$$(w_1/x_1)(\partial x_1/\partial w_1) = \sum_i (x_i^1/x_1)s_i^1s_{i1} + \sum_i \sum_j (x_i^1/x_1)s_i^1\eta_{ij}$$

(A1)

where $s_i^1$ is the cost share of the raw material in the $i$th product and $\eta^i$ is the elasticity of demand for product $i$ with respect to the price of product $j$. The left-hand side of (A1) is the elasticity of demand of $x_1$. Because elasticities of output-constant factor demands are products of cost shares and Allen partial elasticities of substitution, the expression for $s_{11}$ in (15) is obtained by dividing both sides of (A1) by $s_1$.

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