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Competitive Storage, Rational Expectations, and Short-Run Food Price Determination

Michael K. Wohlgenant

This paper demonstrates that lags between retail and wholesale food prices can be explained by inventory behavior of retailers. Theoretical considerations indicate that the markup model should be modified to include a Jorgenson-type user cost variable, which depends on expected future wholesale price. The rational expectations hypothesis is used to derive price expectations. The retail price specification, therefore, depends on the stochastic process generating expected wholesale price. The econometric methodology, employing both causality testing and nonlinear estimation, is illustrated by estimating monthly price relationships for beef. The results are consistent with the theory and indicate rejection of the markup model.

Key words: beef, causality testing, dynamic models, food prices, inventories, marketing margins, rational expectations.

One stylized fact of empirical work on price spreads for food commodities is that, in the short run, retail prices do not adjust instantaneously to changes in wholesale and farm prices (e.g., King, Heien, Luttrell, Lamm and Westcott). A number of explanations for this phenomenon have been provided, but the one cited most often is maintenance of inventories by market middlemen to smooth price fluctuations in response to transitory demand and supply changes (Alchian and Allen, pp. 86–89; Luttrell). Despite its intuitive appeal, this hypothesis has not been subjected to empirical verification. This paper uses the rational expectations framework developed by Sargent (1979, 1981) and Hansen and Sargent to derive testable implications of the role of inventories on the relationship between retail and wholesale food prices.

The role of inventories as a source of price stickiness in response to fluctuations in demand is well documented in the literature (Maccini, Blinder, Amihud and Mendelson). For food commodities, changes in supply account for the majority of short-run price fluctuations. Therefore, the framework needs to be broadened to account for the influence of both demand and supply fluctuations on price behavior. Another consideration is that for higher stages in the marketing system, information on inventories is generally lacking. Thus, it is desirable to have a model which can be used to quantify effects of inventories on price relationships but does not require inventory data directly.

The model proposed here differs from previous specifications in that it focuses on the relationship between the wholesale-retail price spread and the costs of holding inventories. The motivation for this specification is that, in the long run, retail price equals unit raw material costs plus the full marginal costs of processing, distribution, and storage. Under competitive conditions, the cost of inventories equals the market rental rate. In the steady state, when all future prices are the same, the market rental rate equals the interest cost of raw materials, which is the opportunity cost of funds tied up in inventories rather than in an investment earning the current interest rate. In the short run, the market rental rate also depends on anticipated gains or losses from holding inventories. This component of inventory costs can lead to price stickiness and time lags between raw material and final product prices.

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The process by which this occurs can be described as follows. Suppose, for whatever reason, firms expect next period's raw material price to rise relative to this period's price. Since this lowers the relative costs of holding inventories, there is an incentive for firms to buy more of the raw material now and increase their stock holdings. In turn, this increased supply puts downward pressure on retail price so retail price will not rise as much as it would in the absence of inventories. Conversely, when next period's raw material price is expected to decline relative to this period's price, there is an incentive for firms to reduce their inventory holdings, which tends to moderate the initial downward pressure on retail price. In either event, current retail price will not adjust fully to a change in the current raw material price.

This description indicates that the intertemporal linkage is price expectations. This paper models these expectations using the rational expectations hypothesis (REH). The main appeal of the REH is that expectations are viewed as the predictions of the relevant economic theory (Muth). The REH is used here to specify the general form of the price expectations model and to determine the conditions in which rational expectations can be characterized as extrapolative expectations, based solely on current and lagged prices.

The plan of the paper is as follows. The next section puts forth a theory which delineates the role of inventory costs and input price expectations in short-run food price determination. The third section derives the rational price expectations when there are shocks both to demand and to supply. The fourth section illustrates this model by estimating the relationship between wholesale and retail prices for choice beef. The final section offers some concluding remarks. Throughout, linear specifications are employed to exploit the advantages of certainty equivalence and to obtain closed-form solutions for the rational expectations.

The Price Equation with Inventory Costs

Consider a firm who at time t sells a given commodity s_t at a market-determined price P_t; it purchases a quantity q_t at a unit wholesale price W_t; and it holds a beginning inventory level of the amount i_t. For convenience, assume the only costs other than raw material and storage costs are distribution costs. These costs are assumed to be a linear function of the sales rate of the form, aC_ts_t, where a is a row vector of constants and C_t is a column vector of distribution costs (e.g., wage rates, transport costs, packaging costs, etc.). Inventory costs are assumed to have the quadratic form

$$\frac{f}{2}(i_{t+1} - g_0 - gs_t)^2 \tag{1}$$

where f, g_0, and g are parameters; and i_{t+1} is the ending inventory level.[1] Ignoring depreciation, the inventory constraint is[2]

$$i_{t+1} = i_t + q_t - s_t. \tag{2}$$

The quadratic specification of inventory costs, equation (1), is due to Holt et al. This equation can be thought of as the sum of two offsetting costs: (a) physical costs of holding inventories, which rise as the inventory level rises; and (b) costs of stocking out and set-up costs, which for a given sales rate decline as the inventory level rises. (This is because both the probability of stocking out and number of transactions per time period decline as the inventory-to-sales ratio rises.) These costs reach a minimum at some critical inventory level i^*, which depends on the current period sales rate. Thus, as sales change, both total and marginal cost functions shift to reflect changes in back-order and set-up costs. While the coefficients g_0 and g are assumed to be constants, this discussion indicates that they really depend on a deeper set of structural coefficients, including inventory holding and depletion costs, set-up costs, and the second moments of the probability density function of sales (see Holt et al. or Blanchard).

Given these specifications, the firm is assumed to maximize the expected present discounted value of net revenue from inventory holding

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[1] Note that this inventory cost specification assumes negative inventory levels are possible. Following Belsley (chap. 2), this is equivalent to assuming firms treat unmet demand as unfilled orders. For a similar interpretation, see Blinder (p. 337).

[2] With depreciation in product value from storage, equation (2) would be modified as

$$i_{t+1} = i_t + q_t - s_t - \delta i_t$$

where δ is the proportionate rate of depreciation per unit time from storge. This specification says that one unit of the old product is equal to $1 - \delta$ units of the new product. When this equation is used instead of (2) the same first-order conditions (4) and (5) result when the discount factor b is replaced by $b^* = b(1 - \delta)$.

$$(3)\quad E_t \sum_{j=0}^{\infty} b^j[(P_{t+j} - aC_{t+j})s_{t+j} - W_{t+j}q_{t+j} - \frac{f}{2}(i_{t+j+1} - g_0 - gs_{t+j})^2],$$

subject to the inventory constraint (2) with i_t given. Here $E_tZ_t = E(Z_t|\Omega_t)$ is the mathematical expectation conditional on information at time t, Ω_t, and $b < 1$ is the constant discount factor. For the representative firm, the information set is assumed to include i and current and lagged values of P, W, and C. Following Sargent (1979, chap. 14), assume that the stochastic processes involving P, W, and C are each of exponential order less than $b^{-1/2}$ so that the expected present value (3) is some finite number. Using the inventory constraint (2) to eliminate q_t from (3), the firm then maximizes

$$E_t \sum_{j=0}^{\infty} b^j[(P_{t+j} - aC_{t+j} - W_{t+j})s_{t+j} - W_{t+j}(i_{t+j+1} - i_{t+j}) - \frac{f}{2}(i_{t+j+1} - g_0 - gs_{t+j})^2].$$

The first-order conditions for all $j \geq 0$ are

$$(4)\quad E_{t+j}[P_{t+j} - aC_{t+j} - W_{t+j} + fg(i_{t+j+1} - g_0 - gs_{t+j})] = 0,$$

$$(5)\quad E_{t+j}[-W_{t+j} + bW_{t+j+1} - f(i_{t+j+1} - g_0 - gs_{t+j})] = 0,$$

where $E_{t+j}Z_{t+j} = E(Z_{t+j}|\Omega_{t+j})$.

Equation (4) is the standard condition that expected price equals expected marginal cost. Marginal cost equals the sum of average production costs plus a term involving the deviation of actual from the critical inventory level $i^*_{t+i} = g_0 + gs_t$. This term is a component of marginal cost because changes in the sales rate lead to changes in set-up costs and the costs of stocking out. Equation (5) equates expected marginal returns with expected marginal costs of adding inventory. Expected marginal returns are the gap between next period's discounted expected price and this period's price. Expected marginal costs depend on the deviation of actual from the critical inventory level. Equation (5) implies that when there are no speculative gains or losses from holding inventories, expected ending inventory will be a linear function of current period expected sales. When the discounted price is expected to rise (fall), firms will demand more (less) inventories.[3]

Taken together, equations (4) and (5) imply retail price equals full marginal cost, including inventory carrying costs. To see this, solve equation (5) for $E_{t+j}(i_{t+j+1} - g_0 - gs_{t+j})$ and substitute into (4). At $j = 0$ this equation can be written

$$P_t = W_t + aC_t + g(W_t - bE_tW_{t+1}),\text{ or}$$

$$(6)\quad P_t = W_t + aC_t + bg[(b^{-1} - 1)W_t - (E_tW_{t+1} - W_t)].$$

Since $b = (1 + r)^{-1}$ where r is the real interest rate, the term in brackets on the right-hand side of (6) can be interpreted as the user cost of inventory capital (Jorgenson). The first component is the interest cost of the raw material, while the second component represents expected capital gains (losses) from holding inventories.

Equation (6) explains why retail price lags wholesale price. Define $\bar{P}_t = W_t + aC_t + g(1 - b)W_t$. Then equation (6) can be written

$$(7)\quad P_t = \bar{P}_t + bg(W_t - E_tW_{t+1}).$$

This specification indicates that retail price will deviate from its steady-state value $\bar{P}_t$ whenever firms expect next period's wholesale price to differ from the current period price. When next period's price is expected to rise (decline) relative to the current period price, retail price will be below (above) its steady-state value. Because expectations are formed on the basis of current and past information, this implies that any event that causes expected price to diverge from actual price will cause retail price changes to lag wholesale prices.

The main advantage of (6) from an empirical standpoint is that, as opposed to (4), it circumvents the need for inventory data. Another desirable feature of this specification is

[3] As a referee points out, equations (4) and (5) constitute a set of singular equations which cannot be solved for expected sales and expected inventories as functions of output and input prices. The reason is that the production technology is linearly homogenous, which makes production costs a linear function of the rate of output. This assumption does not invalidate the analysis because equations (4) and (5) are used only to characterize equilibrium behavior of the representative firm in the industry, rather than to explain individual firm behavior. These two equations are only part of a more complete system which includes a demand function for the retail product and supply function for the raw material. Thus, at the industry level, prices are not parametric but are determined simultaneously through supply and demand.

that it includes as a special case the standard markup price equation, $P_t = W_t + aC_t$ (see, e.g., Heien). Thus, given the augmented hypothesis of inventory induced price adjustment, it is possible to test the markup pricing hypothesis.

The main obstacle to implementing this model empirically is a model for price expectations. The approach adopted here is rational expectations, where the expectation is taken as the true mathematical expectation of the future price, conditional on the information available to firms at time t. The main advantage of this approach is that it delineates the supply and demand factors thought to be important in the determination of W_t and therefore E_tW_{t+1}. The next section shows that when there is a single shock to the market in the wholesale supply function, the optimal extrapolative predictor coincides with the rational expectation. When there are shocks to both demand and supply, the extrapolative predictor should be augmented to include the effects of anticipated demand shocks on price.

Food Price Determination with Rational Expectations

The relevant economic structure for deriving the rational price expectation consists of specifications for retail demand and wholesale supply in addition to the aggregate behavioral equations for retailer behavior, equations (4) and (5).[4] These equations can be written,

$$(8) \quad S_{t+j} = -dP_{t+j} + \delta_{t+j}, \quad \text{(retail demand)}$$

$$(9) \quad P_{t+j} - W_{t+j} - aC_{t+j} + kg(E_{t+j}I_{t+j+1} - gE_{t+j}S_{t+j}) = 0, \quad \text{(wholesale-retail price spread)}$$

$$(10) \quad -W_{t+j} + bE_{t+j}W_{t+j+1} - k(E_{t+j}I_{t+j+1} - gE_{t+j}S_{t+j}) = 0, \quad \text{(Inventory demand)}$$

$$(11) \quad Q_{t+j} = eW_{t+j} + \sigma_{t+j}, \quad \text{(wholesale supply)}$$

$$(12) \quad I_{t+j+1} = I_{t+j} + Q_{t+j} - S_{t+j} \quad \text{(market clearing)}$$

where $k = N^{-1}f$ (N = number of firms). Here capital letters denote industry-wide magnitudes. The parameters δ_{t+j} and σ_{t+j} are exogenous shocks to retail demand and wholesale supply, respectively. The parameter g_0 has been deleted from the retail behavioral equations (9) and (10) because it serves no useful purpose in the ensuing analysis. The information set available to firms is assumed to include I_t, current and lagged values of C_t and σ_t, but only lagged values of δ_t. This is consistent with the view that, at the time decisions are made, firms observe fluctuations in supply but not in demand. Also note that the wholesale supply equation (11) is specified as a function of actual, rather than expected, price. This specification could be changed to depend on expected price, but this would not affect the results in any substantive way.

By the rational expectations hypothesis,

$$E_{t+j}I_{t+j+1} = I_{t+j+1} - U_{1t+j} \text{ and}$$
$$E_{t+j}S_{t+j} = S_{t+j} - U_{2t+j}$$

where U_{1t+j} and U_{2t+j} are uncorrelated with information available to firms at time $t + j$. This means at time $t + j$, (8)–(12) can be thought of as a system of five equations in the five endogenous variables: $E_{t+j}S_{t+j}$, P_{t+j}, E_tI_{t+j+1}, Q_{t+j} and W_{t+j}. With negatively sloped demand and positively sloped supply curves, this system ordinarily will yield unique solutions for these five endogenous variables for given values of I_{t+j}, $E_{t+j}\delta_{t+j}$, σ_{t+j}, and C_{t+j}.

The rational expectations equilibrium can be obtained by solving (8)–(12) for the reduced form, and by then reducing this set of equations to a single, second-order difference equation in W_{t+j}. First, solve (8) for P_{t+j} and substitute this result into (9) to obtain

$$(13) \quad E_{t+j}S_{t+j} = \alpha(kgE_{t+j}I_{t+j+1} + d^{-1}E_{t+j}\delta_{t+j} - W_{t+j} - aC_{t+j})$$

where $\alpha = d/(1 + dkg^2)$. Next, use (11) and (13) to eliminate $E_{t+j}S_{t+j}$ and Q_{t+j} from (10) and (12). This results in the system of first-order difference equations

$$(14) \quad bE_{t+j}W_{t+j+1} - a_{11}W_{t+j} - a_{12}E_{t+j}I_{t+j+1} = -\alpha kg(d^{-1}E_{t+j}\delta_{t+j} - aC_{t+j}),$$

$$(15) \quad a_{11}E_{t+j}I_{t+j+1} - I_{t+j} - i_{21}W_{t+j} = \sigma_{t+j} - \alpha(d^{-1}E_{t+j}\delta_{t+j} - aC_{t+j})$$

where $a_{11} = (1 + \alpha kg)$, $a_{12} = (k/d)\alpha$, and $a_{21} = (\alpha + e)$. Finally, equations (14) and (15) can be reduced to a single, second-order difference equation in $E_{t+j}W_{t+j+1}$:

[4] Equations (4) and (5) are for the representative firm in the industry. The aggregate behavioral equations are obtained by multiplying all average quantities by the number of firms in the industry.

$$(16)\quad (1 - \phi B + b^{-1}B^2)E_{t+j}W_{t+j+1}$$
$$= (ba_{11})^{-1}a_{12}E_{t+j}\sigma_{t+j} - (ba_{11})^{-1}[(a_{12} + kga_{11}) - kgB]\alpha(d^{-1}E_{t+j}\delta_{t+j} - aE_{t+j}C_{t+j})$$

where $\phi = (ba_{11})^{-1}(b + a_{11}^2 + a_{12}a_{21})$. Here B is the backward shift operator such that $BE_tZ_{t+1} = E_tZ_t$.

To solve (16) follow Sargent (1979, chap. 14) and obtain the factorization

$$(1 - \phi B + b^{-1}B^2) = (1 - \lambda_1 B)(1 - \lambda_2 B),$$

implying

$$\phi = \lambda_1 + \lambda_2 \text{ and } \lambda_1\lambda_2 = b^{-1}.$$

It is straightforward to verify that these roots are real and distinct such that (see Sargent 1979, p. 198)

$$0 < \lambda_1 < 1 < b^{-1} < \lambda_2.$$

Noting that $(1 - \lambda_2 B) = -(1 - \lambda_2^{-1}B^{-1})\lambda_2 B$ and $\lambda_2^{-1} = \lambda_1 b$, the solution to (16) at $j = 0$ can be expressed as

$$(17)\quad (1 - \lambda_1 B)E_tW_{t+1} = -\lambda_1 a_{11}^{-1}a_{12}(1 - \lambda_1 bB^{-1})^{-1}E_t\sigma_{t+1} + \lambda_1 a_{11}^{-1}(1 - \lambda_1 bB^{-1})^{-1}[(a_{12} + kga_{11}) + kgB]\alpha(d^{-1}E_t\delta_{t+1} - aE_tC_{t+1}).$$

Thus the rational expectation of next period's wholesale price, W_{t+1}, can be expressed as a linear function of the current period price and conditional expectations of all future shocks to supply and demand. This implies by equation (6) that the relationship between retail and wholesale prices depends on retailers' expectations of all future shocks to supply and demand.

To make the empirical implications of the rational expectations hypothesis concrete, consider first the case where the only shock to the market is in the wholesale supply equation. Suppose, for the sake of argument, the supply shock follows the simple first-order Markov process

$$(18)\quad \sigma_t = \psi\sigma_{t-1} + \epsilon_t$$

where $|\psi| < 1$ and ϵ_t is i.i.d. with mean zero and constant variance. Since $E_t\epsilon_{t+j} = 0$ for all $j \geq 1$ it follows that $E_t\sigma_{t+j} = \psi^j\sigma_t$ for $j \geq 1$. Using this result in equation (17) yields

$$E_tW_{t+1} = \lambda_1 W_t - \lambda_1 a_{11}^{-1}a_{12}(1 - \lambda_1 b\psi)^{-1}\psi\sigma_t.$$

Substituting this solution for E_tW_{t+1} in (16) when $j = 0$ the reduced-form solution for W_t can be shown to equal

$$(19)\quad W_t = \lambda_1 W_{t-1} - \lambda_1 a_{11}^{-1}a_{12}(1 - \lambda_1 b\psi)^{-1}\sigma_t.$$

Multiplying both sides of this equation by $(1 - \psi L)$, where L is the lag operator, gives the second-order autoregressive process

$$(20)\quad W_t = (\psi + \lambda_1)W_{t-1} - \psi\lambda_1 W_{t-2} + \eta_t,$$

where $\eta_t = -\lambda_1 a_{11}^{-1}a_{12}(1 - \lambda_1 b\psi)^{-1}\epsilon_t$. Shifting equation (20) forward by one time period and taking conditional expectations of both sides yields

$$(21)\quad E_tW_{t+1} = (\psi + \lambda_1)W_t - \psi\lambda_1 W_{t-1}.$$

Thus for this stochastic specification the extrapolative predictor coincides with the rational expectation. In this special case, the optimal predictor is a second-order autoregressive process. For more general stochastic processes, the optimal predictor can be derived as the one step-ahead forecasts from an ARMA specification of W_t or, when the moving-average component is invertible, as an infinite autoregressive process (see, e.g., Wallis). In any event, when there is a single shock to the market, the rational expectation can be characterized as an extrapolative predictor, based solely on current and lagged wholesale prices.

When there are shocks to both supply and demand, extrapolative expectations are no longer optimal. This point can be made most forcefully by considering an expanded model in which supply shocks are generated by the first-order scheme in (18), and in which demand shocks follow the first-order process

$$(22)\quad \delta_t = \rho\delta_{t-1} + \mu_t,$$

where $|\rho| < 1$ and μ_t is i.i.d. with mean zero and constant variance.[5] Using (18) and (22) in (17), the rational expectation of W_{t+1} now can be characterized as

$$E_tW_{t+1} = \lambda_1 W_t - \lambda_1 a_{11}^{-1}a_{12}(1 - \lambda_1 b\psi)^{-1}\psi\sigma_t + \lambda a_{11}^{-1}[(a_{12} + kga_{11}) - kg\rho^{-1}]\alpha d^{-1}(1 - \lambda_1 b\rho)^{-1}\rho^2\delta_{t-1}.$$

Using this result in (16) yields the counterpart to equation (19) as

$$W_t = \lambda_1 W_{t-1} + A_1\sigma_t + A_2\delta_{t-1}$$

[5] Marketing costs are not included in this specification to simplify the argument.

where the A's are functions of the structural coefficients. Multiplying both sides of this equation by $(1 - \psi L)$ then gives

$$(23)\quad W_t = (\psi + \lambda_1)W_{t-1} - \psi\lambda_1 W_{t-2} + A_2(1 - \psi L)\delta_{t-1} + \eta_t,$$

which is equation (20) augmented by the terms involving δ_{t-1} and δ_{t-2}. Shifting this equation forward one time period and taking conditional expectations of both sides results in the augmented rational expectation model,

$$(24)\quad E_t W_{t+1} = (\psi + \lambda_1)W_t - \psi\lambda_1 W_{t-1} + A_2(\rho - \psi L)\delta_{t-1},$$

which shows that, in general, the extrapolative expectation (21) will be suboptimal when firms respond to both supply and demand shocks. This point was made in related contexts by Nelson and by Wallis. The contribution of the present analysis is to demonstrate that the price expectations model can be viewed as the sum of two components: (*a*) changes from anticipated supply fluctuations, which can be represented (optimally) by current and lagged wholesale prices; and (*b*) changes from anticipated demand shocks, which can be represented by lagged exogenous demand shifters such as lagged values of income.

Another important aspect of this decomposition is that it suggests a test for extrapolative expectations based on the concept of Granger causality. To see this, substitute equation (24) into equation (6). Aside from marketing costs, this retail price specification can be written

$$(25)\quad P_t = B_1 W_t + B_2 W_{t+1} + B_3(1 - \psi L)\delta_{t-1}$$

where the B's are functions of the structural coefficients. Rewrite equation (23) as

$$(26)\quad W_t = C_1 W_{t-1} + C_2 W_{t-2} + C_3(1 - \psi L)\delta_{t-1} + \eta_t$$

where $C_1 \equiv (\psi + \lambda_1)$, $C_2 \equiv -\psi\lambda$, and $C_3 \equiv A_2$. Since $|\psi| < 1$, equation (25) can be solved for δ_{t-1}. Substituting this result into (26) and solving for W_t gives

$$(27)\quad W_t = D_1 W_{t-1} + D_2 W_{t-2} + D_3 P_t + \eta'_t$$

where

$$D_1 = (B_3 + B_1C_3)^{-1}(B_3C_1 - B_2C_3),$$
$$D_2 = (B_3 + B_1C_3)^{-1}B_3C_2,$$
$$D_3 = (B_3 + B_1C_3)^{-1}C_3, \text{ and}$$
$$\eta'_t = (B_3 + B_1C_3)^{-1}B_3\eta_t.$$

Next, use (22) to eliminate δ_{t-1} and δ_{t-2} from (25) and substitute (27) into this result to obtain

$$(28)\quad P_t = (\rho - \psi)P_{t-1} + \psi(\rho - \psi)P_{t-2} + \psi^2(\rho - \psi)P_{t-3} + \ldots + [B_1D_1 + (B_2 - \rho B_1)]W_{t-1} + (B_1D_2 - B_2\rho)W_{t-2} + B_3\mu_{t-1}.$$

Finally, substitute equation (28) into (27) to get

$$(29)\quad W_t = D'_1 W_{t-1} + D'_2 W_{t-2} + D_3(\rho - \psi)P_{t-1} + D_3\psi(\rho - \psi)P_{t-2} + D_3\psi^2(\rho - \psi)P_{t-3} + \ldots + (\eta'_t + D_3B_3\mu_{t-1}),$$

where

$$D'_1 = D_1 + D_3[B_1D_1 + (B_2 - \rho B_1)] \text{ and}$$
$$D'_2 = D_2 + D_3(B_1D_2 - B_2\rho).$$

Equations (28) and (29) constitute the bivariate autoregressive representation of (P_t, W_t) for the stochastic specifications (22), (25), and (26). Equation (29) indicates that whether past demand changes help to explain current wholesale price is equivalent to testing for the significance of lagged retail prices, save for the case when $\rho = \psi$. But this is equivalent to testing the null hypothesis that P fails to Granger-cause W. Thus, failure to reject this hypothesis can be taken as empirical support for an extrapolative expectations model. If the null hypothesis is rejected, the price expectations model should be augmented to include the effects of demand shocks.

As pointed out by Hansen and Sargent, one-way Granger causality from W to P does not ensure that W_t is strictly exogenous with respect to P_t. The reason is that the triangular representation of (P_t, W_t) is only a necessary but not sufficient condition for recursiveness. In other words, contemporaneous correlation between the innovations in P_t and W_t in general cannot be ruled out. As indicated by Hansen and Sargent, instantaneous causality can occur when firms observe and respond to more information than the econometrician. It can also occur when demand shocks coincide with supply shocks. This can be seen from equation (29), which indicates that when $\rho = \psi$ lagged P's do not contribute to W, even though by (27) W and P are still jointly determined. This means that an autoregressive specification like (20), even if it is found to be a workable approximation for the data, is best viewed as a reduced-form equation relating wholesale price to both lagged supply and demand changes. It also means that the retail and wholesale price equations should be fitted jointly with the cross-equation restrictions imposed in order to disentangle the structural coefficients of the retail price equation (6) from the stochastic process generating W_t.[6] The next section shows how this econometric methodology can be implemented by estimating the relationship between wholesale and retail prices for choice beef.

Empirical Application

This section uses the econometric methodology developed in the paper to estimate the relationship between monthly wholesale and retail prices for beef. Price data are for choice yield grade 3 beef, and wholesale price is the retail equivalent of net carcass value. These data were obtained from selected issues of *Livestock and Poultry Outlook and Situation Report* and cover the period January 1976 through October 1983. The first four months were used to compute the initial lags, leaving a total of ninety observations. Prices were deflated by the monthly consumer price index. An intercept, a set of eleven monthly dummies, and a linear time trend were also included in each regression.

The first step in formulating the model is to see if rational forecasts of next period's wholesale price can be formed solely on the basis of current and past prices. As shown in the previous section, a necessary condition for this to hold is that current and future retail prices fail to Granger-cause wholesale price, i.e., lagged P's fail to predict W. The null hypothesis that lagged retail prices do not help to predict current wholesale price yielded an F-statistic of 2.42 with 4 and 69 degrees of freedom, implying this hypothesis cannot be rejected at slightly greater than the 5% significance level. This test, therefore, provides preliminary evidence for the extrapolative price expectation model.[7]

Given this result, the retail and wholesale price specifications entertained were

$$(30)\quad M_t = g(1 - bb_1)W_t - gbb_2W_{t-1} - gbb_3W_{t-2} - gbb_4W_{t-3} + \epsilon_{1t},$$

$$(31)\quad W_t = b_1W_{t-1} + b_2W_{t-2} + b_3W_{t-3} + b_4W_{t-4} + U_{2t},$$

where $M_t = P_t - W_t$ (wholesale-retail price spread), and ϵ_{1t} and U_{2t} are random disturbance terms. Equation (30) was derived by substituting the conditional one-step-ahead forecasts for W_{t+1} from (31) into equation (6).[8] A wage rate variable (deflated wage rate in grocery stores) was initially included, but dropped because its effect was insignificant. Monthly intercepts and a trend variable were included in each equation to account for systematic changes in the price spread and wholesale price but are not written here to conserve space. Preliminary analysis also indicated serial correlation in the price spread equation, so ϵ_{1t} was assumed to follow the first-order process

$$(32)\quad \epsilon_{1t} = \theta\epsilon_{1t-1} + U_{1t}$$

where $|\theta| < 1$.[9] Using (32) to eliminate ϵ_{1t} from (30), the specification for M_t can be written

$$(33)\quad M_t = \theta M_{t-1} + g(1 - bb_1)W_t - [gbb_2 + \theta g(1 - bb_1)]W_{t-1} - gb(b_3 - \theta b_2)W_{t-2} - gb(b_4 - \theta b_3)W_{t-3} + \theta gbb_4W_{t-4} + U_{1t}.$$

By the analysis of the previous section, contemporaneous correlation between U_{1t} and U_{2t} in general cannot be ruled out. This implies equations (31) and (33) should be estimated jointly. The estimation technique used was Gallant's seemingly unrelated nonlinear regressions method, which is asymptotically equivalent to maximum likelihood. This method was implemented by first deriving the reduced-form equations, and then estimating the unrestricted and restricted reduced forms

[6] Wu shows that causality tests are neither necessary nor sufficient for establishing predeterminedness. This point was made in a related context by Conway et al. See their article for additional caveats associated with causality testing. It should be pointed out that contemporaneous correlation between P_t and W_t in no way is influenced by the specific information assumptions made. Specifically, a wholesale price specification with the same stochastic properties would result even if supply shocks are not contemporaneously observable.

[7] Since there is reason to believe the error terms of P and W are contemporaneously correlated, the causality test also was conducted by comparing the residual sum of squares of the unrestricted and restricted models estimated by the joint generalized least-squares method (Theil, chap. 7). The results were the same, indicating P fails to Granger-cause W.

[8] The reason the price spread, rather than retail price, formulation is used here is for computational convenience. Note that both specifications will yield the same parameter estimates because, by construction of the data, retail price is identically equal to the price spread plus wholesale price.

[9] Hansen and Sargent point out that a plausible model for the error term is information observed by firms but not observed by the econometrician. This suggests the error term will be serially correlated. In this case it seems plausible that ϵ_{1t} reflects the influence of unobserved marketing costs, which are likely autocorrelated.

subject to the same contemporaneous variance-covariance matrix estimated from the unrestricted model. Aside from the monthly intercepts and linear trend, there are a total of nine reduced-form parameters and seven free parameters of the model (θ, g, b, b_1, b_2, b_3, b_4). Thus the model imposes two over-identifying restrictions on the free parameters.

Estimates of the unrestricted and restricted reduced forms are reported in table 1. For most of the parameter values, there is a close correspondence between the unrestricted and restricted estimates. The compatibility of the restrictions with the data were tested using the analog of the likelihood ratio test for the seemingly unrelated nonlinear regressions method (Burguette, Gallant, and Souza). This test was conducted by subtracting the system residual sum of squares for the unrestricted model from the system residual sum of squares for the restricted model, and comparing this number with the tabled chi-squared value with two degrees of freedom. For this model, the observed chi-squared value is 4.61, implying the null hypothesis cannot be rejected at about the 10% significance level. Examination of the estimated correlograms of the residuals of M_t using the Q-statistic of Box and Pierce did not reveal any further evidence of serial correlation. For twenty-four lags, all estimated autocorrelations were within two standard errors and the Q-statistic was 4.46. Thus the restrictions implied by the theory appear to be consistent with the data.

Estimates of the structural parameters are reported in table 2. The two main parameters of interest, b and g, each have the correct sign and are significantly different from zero. An estimate for b of 0.89 with monthly data may seem too small. However, for a perishable commodity like beef, the discount factor should reflect depreciation from storage as well as the time value of money (see note 2). The parameter g can be interpreted as the desired inventory-to-sales ratio (Holt et al., Blanchard). In the case of beef, an estimate for g of 1.97 with monthly data seems too large, but this is hard to prove because published inventory data do not include inventories held in coolers.

Table 2. Structural Parameter Estimates of Price Spread and Wholesale Price Equations for Choice Beef

Parameter	Estimate
b	0.886 (0.068)[a]
g	1.972 (0.910)
θ	0.730 (0.064)
b_1	1.130 (0.080)
b_2	−0.128 (0.051)
b_3	−0.033 (0.029)
b_4	−0.023 (0.027)

Note: Estimated parameters for monthly intercepts and time trend not reported.
[a] Estimated asymptotic standard errors.

Table 1. Restricted and Unrestricted Estimates of Reduced-Form Price Spread and Wholesale Price Equations for Choice Beef

Explanatory Variable	Price Spread, M_t		Wholesale Price, W_t	
	Unrestricted	Restricted	Unrestricted	Restricted
M_{t-1}	0.729 (0.067)[a]	0.730 (0.064)		
W_{t-1}	0.181 (0.086)	0.223 (0.065)	1.193 (0.115)	1.130 (0.080)
W_{t-2}	−0.101 (0.124)	−0.105 (0.072)	−0.141 (0.177)	−0.128 (0.051)
W_{t-3}	0.151 (0.122)	−0.002 (0.065)	−0.275 (0.177)	−0.033 (0.029)
W_{t-4}	−0.149 (0.078)	−0.029 (0.032)	0.178 (0.110)	−0.023 (0.027)

Note: Estimated parameters for monthly intercepts and time trend not reported.
[a] Estimated asymptotic standard error.

The markup pricing hypothesis is a special case of equation (33) with b and g set equal to zero. This hypothesis was tested by reestimating the reduced form with this added restriction imposed and comparing the residual sum of squares of this model with the residual sum of squares of the restricted model with b and g left unconstrained. This test yielded a chi-squared value of 29.76 with 2 degrees of freedom, implying the markup pricing hypothesis can be rejected at a significance level smaller than 0.001.[10] Thus, not only are the data consistent with the theory proposed here, but this specification appears to offer explanatory power above that provided by the markup pricing hypothesis.

Concluding Remarks

The theoretical framework and findings of this paper have a number of implications for modeling short-run food price behavior. First, the rational expectations hypothesis (REH) indicates the form of the estimating equation for retail price (or the wholesale-retail price spread) will depend on the nature of the stochastic process generating next period's wholesale price. In the application presented here, causality tests indicated that expectations could be characterized by an extrapolative predictor, but in other applications a more general expectations model, incorporating the effects of lagged demand variables, might prove superior. Second, the REH imposes over-identifying restrictions on the model. Thus the compatibility of the particular expectations model with the data can be checked prior to empirical implementation. Third, by imposing overidentifying restrictions on the model, the REH makes it possible to disentangle the structural parameters of the price equation from the stochastic process generating the expected price variable. As emphasized by Sargent (1981), this can lead to improved forecasts because we are able to purge the reduced-form coefficients of structural change caused by the effect of policy changes on price expectations. Finally, the theoretical framework could be enlarged to include the influence of other factors, such as the costs of changing inventories. Such specifications would seem plausible and would lead to materially different, though more complicated, restrictions on the behavioral processes. Nonetheless, the empirical application to beef suggests that the simple price specification employed here is a useful starting point for more complicated formulations.

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[10] A referee points out that the markup model can result when either g or b are equal to zero. Therefore, the test used here overstates rejection of this model. However, based on the t-values in table 2, the markup model is still rejected even if b or g are considered separately.

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