

# Positively Dependent Productivity Shocks in Tournaments: An Empirical Analysis of Production Contracts Settlement Data

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## Abstract

In this paper we estimate an empirical cardinal tournament model in which the players' productivity shocks are positively dependent. Our specification, based on the concept of affiliation, is more general than the standard linear additive and independent specification predominantly used in the literature. We apply our method to modeling the contract settlement data for the production of broiler chickens. The results show that our specification fits the data better and yields different predictions about the magnitude and distribution of benefits (profits) and costs of contractual relations. As these contracts are frequently challenged in courts and are also a target of continuous regulatory proposals, these findings have very important legal and policy ramifications.

**Keywords:** Affiliation, Copulas, Contracting, Cardinal Tournaments.

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# 1 Introduction

Tournaments are relative performance labor contracts where an individual employee's compensation depends on his/her own performance relative to others. The use of tournaments is closely tied to solving the problem of moral hazard in teams or groups. In a typical setting, the observed contracted outcome (output) that the principal's payoff (profit) depends upon is influenced by the agents' unobservable effort and some shock. Using payment schemes that depend on observed outcome, contracts provide incentives for agents to exert unobservable effort. However, being risk-averse, agents want to be insured against large fluctuations in their income. But, complete insurance surely invites shirking. In the presence of production shocks that are common to all agents, the principal may be able to offer some insurance to agents if their obtained outcomes convey information about that common shock. In this context, relative performance evaluation via tournaments provides a mechanism to partially insure agents by filtering away that common shock. Contest among agents have no intrinsic value in improving agents performance, they are only valuable to the extent that peer performance offers information about the common shock (e.g. Lazear and Rosen 1981; Holmström 1982; Green and Stokey 1983; Nalebuff and Stiglitz 1983; Tsoulouhas and Vukina 1999).

In this paper we use the payroll data from a contract for the production of broiler chickens. Broiler production contracts is a well known and documented example of the use of tournaments in real business setting (e.g. Knoeber and Thurman 1994; Tsoulouhas and Vukina 1999; Levy and Vukina 2004). The examples of common production uncertainties in the broiler industry that make tournaments particularly useful include the effects of weather (temperature and humidity), untried feed rations, newly introduced genetic strains, outbreaks of contagious diseases, etc. In most of the tournaments literature, the productivity shock an agent faces during the production process is decomposed using linear additive specification in the following form:

$$u_{it} = \eta_t + v_{it} \tag{1}$$

where  $u_{it}$  is the productivity shock to agent  $i$  in tournament  $t$ ,  $\eta_t$  is the common shock in

tournament  $t$ ,  $v_{it}$  is the idiosyncratic shock of agent  $i$  in tournament  $t$  and  $\eta_t$  and  $v_{it}$  are independent from each other (e.g. Levy and Vukina 2004; Vukina and Zheng 2007; Zheng and Vukina 2007; Vukina and Zheng 2009).<sup>1</sup> This specification is appealing because, among other things, it facilitates a direct welfare comparison of relative performance compensation schemes such as tournaments with absolute performance schemes such as piece rates. Because tournaments eliminate the influence of common production shocks from the agents' wage but impose the group composition risk, whereas the piece rates do exactly the opposite, the optimal choice between the two schemes depends on the relative magnitudes of these two effects.<sup>2</sup>

However, this linear additive and independent specification of stochastic production technology is quite restrictive. The main purpose of this paper is to propose an alternative and more general way of introducing the common shock component into the productivity shock and then to examine its implications. More specifically, we specify the productivity shocks to different agents in the same tournament to be positively dependent with one another. To model positive dependency of productivity shocks we use the concept of affiliation, originally introduced by Milgrom and Weber (1982). Since its introduction, affiliation has become an important concept in the development of the auction theory used to formalize the assumption that latent valuations of potential bidders are probably dependent in some way. However, surprisingly, to date, it has been ignored in the tournament literature, despite the fact that auction and tournament models have many characteristics in common.

The statistical concept of affiliation describes the positive dependence structure between random variables, implying that a high value of one variable makes high values of other variables more likely than small values. In case of broiler production, in summer months, for example, when the weather is hot and humid, chickens waste energy on cooling and

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<sup>1</sup>In some of these papers, the authors use a multiplicative specification  $u_{it} = \eta_t v_{it}$ , which is essentially the same because taking logs on both sides of the multiplicative specification yields linear additive specification.

<sup>2</sup>Levy and Vukina (2004) define the group (league) composition effect as the random variable  $y_{it}^n - y_{it}^\infty$ , where  $y_{it}^\infty$  is a stochastic payment of agent  $i$  in tournament  $t$  when the tournament league consists of the entire population of contestants and  $y_{it}^n$  is a payment when the league consists of  $n$  contestants randomly drawn from the population.

convert feed into weight gain very inefficiently, hence hot and humid weather is considered to be a bad production shock. However, since growers who compete in the same tournament lives in close proximity and grow flocks at the same time, if it was hot and humid on one farm, the same was true for the other farm, so their shocks are affiliated. The concept of affiliation could be useful in both auctions and tournament settings because it preserves the covariation under some arbitrary monotone transformation of the random variables. This is not the case with correlation which measures only the linear dependency. Then, in cases when this linearity is destroyed with some monotone transformation, the correlation can possibly show no relationship between variables, even if the dependency still exist.

To model the positive dependence among the agents' productivity shocks, we adopt the *copula* approach. Nelsen (1999) has provided a detailed introduction to the theory of copulas. Copulas, which provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions, have seen growing applications first in actuarial science (e.g., Frees and Valdez 1998) and more recently in economics. For example, copula approach has been recently used to characterize the joint yield and price risk in agricultural crops with the purpose of calculating premium rates for whole farm (revenue) insurance (Zhu, 2009). Also, several papers have used copulas to model positive dependence in IO applications, e.g. Hubbard, Li and Paarsch (2009), Li and Zhang (2009b), and Miravete (2009).

This paper also contributes to the growing literature on structural econometrics approach to estimating tournament models, which proves to be quite useful for conducting welfare analyses. Recently, several papers have estimated structural models of various type tournaments. Ferrall and Smith (1999) estimated a sequential tournament game for championship series in sports. Ferrall (1996) and Chen and Shum (2009) estimated elimination tournament models for workers competing for limited promotion slots. Zheng and Vukina (2007) estimated a rank-order tournament model to quantify the efficiency gains of an organizational innovation that would replace an ordinal tournament with a cardinal one. Vukina and Zheng (2009) estimated a piece-rate tournament model to quantify the welfare effects of group heterogeneity. Our work is also related to the literature on estimating and testing auctions models in

which bidders' private values are affiliated. Examples include Li, Perrigne and Vuong (2000, 2002), Hubbard, Li and Paarsch (2009), de Castro and Paarsch (2008), Jun, Pinkse and Wan (2008) and Li and Zhang (2009a, 2009b).

We find that our model is strongly preferred by the data to the standard specification of decomposing the productivity shock with a linear additive specification. More importantly, we find that these two specifications have different welfare implications because they yield different optimal contract parameters and consequently different predictions about equilibrium effort and different predictions about the cost and benefits of contractual relations. For instance, the simulation results show that the standard linear additive specification of production shocks tends to underestimate the principal's average expected total cost by about 7.85% relative to the affiliation approach. Consequently, the standard specification of productivity shocks would systematically overestimate the integrators profits.

The rest of the paper is organized as follows. In the next section we describe the essential features of broiler production contracts and introduce the data set. In Section 3 we introduce the theoretical model of the cardinal tournament. Section 4 is devoted to the estimation methodology and the presentation of results. In Section 5, we conduct simulation analyses and in Section 6 we conclude. Technical proofs are collected in the Appendix.

## 2 Industry and Data

The poultry industry is often considered a role model for the industrialization of agriculture. The industry is entirely vertically integrated from breeding flocks and hatcheries to feed mills, transportation divisions and processing plants. The final (finishing) stage of production where one day old chicks are brought to the farm and then grown to market weight broilers is organized almost entirely via contracts between companies and independent growers. In the United States, large national companies called integrators, such as Tyson Foods, Pilgrim's Pride, or Perdue Farms dominate broiler contract production. These companies run their operations through smaller divisions spread throughout the country, with heavy concentration

of the production facilities in the south and south-east.

Modern broiler production contracts are agreements between an integrator company and growers that bind farmers to tend for company's chickens until they reach market weight by strictly following specific production practices in exchange for monetary compensation. According to a typical contract, the grower provides land, housing facilities, utilities (electricity and water) and labor and pays for operating expenses such as repairs and maintenance, clean-up, and manure and mortality disposal. The company provides chicks, feed, medication, and the services of field men. Most of the modern broiler contracts are settled using a two-part piece-rate tournament. In this type of tournament, the total payment  $R_i$  to grower  $i$  is the sum of the base rate and the bonus rate multiplied by the live pounds of poultry moved from the grower's farm:

$$R_i = \left[ b + \beta \left( \frac{1}{N} \sum_{j=1}^N \frac{c_j}{y_j} - \frac{c_i}{y_i} \right) \right] y_i, \quad (2)$$

where  $c_i$  is the settlement cost obtained by adding integrator's side customary flock costs (chicks, feed, medication, etc.),  $y_j$  is the weight (measured in pounds) of live chickens produced. As seen from (2), individual grower's piece rate per pound of live poultry produced is the sum of a constant base rate  $b$  (e.g., 3.5 - 4.5 cents a pound), and a variable bonus rate determined by the grower's relative performance. The bonus rate is determined as a percentage  $\beta$  of the difference between group average performance  $\frac{1}{N} \sum_{j=1}^N \frac{c_j}{y_j}$  and the producer's individual performance  $\frac{c_i}{y_i}$ . The performance is measured by  $x_i = \frac{c_i}{y_i}$ , that is, the settlement cost per pound of live chicken produced. The calculation of the group average performance includes all  $N$  growers whose flocks are settled on the same date. In order for all growers to be exposed to the same common production shock, tournaments are usually settled once a week. For the below average settlement cost per pound of chicken produced (above average performance), the grower receives a bonus and for the above average settlement cost per pound of chicken produced, she receives a penalty.

As is explained in Tsoulouhas and Vukina (1999), poultry tournaments are double-margin

contests about who can produce more output (live poultry) with the smallest possible settlement cost. The growers' effort (husbandry practices) stochastically influence the settlement costs (feed utilization) and the quantity of output. Growers can economize with feed (and hence settlement costs  $c_i$ ) by preventing spillage through proper maintenance of feeders and storage bins and by maintaining a housing environment that is conducive to efficient feed conversion. Growers can also separately influence output (live poultry weight  $y_i$ ) by undertaking actions aimed at preventing excessive animal mortality. Depending on the size of the chickens grown, the grow-out process lasts about 7-8 weeks.

Different companies, or different profit centers within the same company, typically specialize in the production of a particular size (weight) birds and offer their own contracts to their growers. The contracts for growing different size birds usually differ only with respect to the base rate (parameter  $b$  in (2)) in that farmers growing heavier birds typically receive larger base rate than those growing smaller birds. Broiler contracts are predominantly short-term (one flock of birds at a time) and explicitly uniform such that all growers, growing the same size birds for the same profit center, receive an identical contract regardless of their past performance, the length of tenure with the company, or any other grower characteristic. The composition of the tournaments (settlement groups) is predominantly governed by timing and logistics of the production process and not with an attempt to form more homogenous or more diverse groups of contestants.

The data set used in this study includes broiler production information gathered from the payroll data of one company's profit center whose production contract corresponds to the payment scheme described in (2).<sup>3</sup> Each observation in the data set represents one contract settlement, i.e., the payment received and the grower performance associated with one grower and one flock of birds delivered to the integrator's processing plant. The data comes from the so called settlement sheets and contain the information on the quantities and costs of various inputs supplied by the integrator (chicks, feed, medication, vaccination etc.), the number of

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<sup>3</sup>The institutional features of the piece-rate contract that generated this data set are remarkably similar to production contracts used by the entire industry. For a detailed description of the industry organizational structure see Vukina (2001).

birds placed and harvested, the quantity of broiler meat (live weight) produced, the dates when production started and terminated, mortality rates, etc.

The settlement dates range from July 1995 to July 1997 totalling 104 tournaments (one tournament per week). The total number of growers is 356 and the total number of usable observations is 3,247 flocks. The average live weight of the fully grown broilers is 4.81 pounds, with a maximum of 5.75 pounds and a minimum of 3.88 pounds, the average number of days that a grower needs to grow chickens to that weight is 53, with a maximum of 79 and a minimum of 43, and the average feed conversion ratio (pounds of feed necessary to produce one pound of live animal weight gain) is 2.03, with a maximum of 3.38 and a minimum of 1.83. The variable piece rate ranges from 2.4 cents to 5.3 cents per pound. Also, the average settlement cost per pound of chicken produced is 31.26 cents, with a standard deviation of 3.22 cents, a maximum of 52.60 cents and a minimum of 24.11 cents.

The participation ranges between 1 and 12 times with an average participation of 9.12 times. The mode of the distribution is 11 as 147 growers participated in 11 tournaments. Given the fact that maximum participation is 12, then the participation rate can be calculated as  $3247/(12*356)$  or 76%. Another way to look at the participation rates is to look at the lengths of the down-time periods (the number of days between the settlement of a previous tournament and the start of the next one). This gap can be as short as 3 days and as long as 109 days. On average this gap is 16.2 days and the median is 12 days. An obvious reason for why some growers participated in fewer tournaments is either because they join the profit center or left the business during the time period covered by our data set. For cases that do not fit this explanation, the differences in participation rates can be explained by the differences in times individual growers need to grow birds to market weight and to clean and prepare the chicken houses for new flocks, and also the integrator's idiosyncrasies in scheduling the delivery of new chicks.<sup>4</sup>

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<sup>4</sup>The random composition of tournament groups in broiler contracts was empirically confirmed by Levy and Vukina (2004). They found that the original tournament groups disintegrate rapidly with typically less than half of the original group remaining intact for the very next tournament. After about four tournaments the groups have largely turned over and subsequent tournaments strongly resemble the random case.

### 3 The Model

The exact modeling of a tournament game that would simultaneously take into account both the feed margin and the output margin is obviously quite complex, if not impossible, both in terms of theoretical modeling as well as econometric estimation. Therefore, following the earlier literature on broiler tournaments, such as Knoeber and Thurman (1994), Tsoulouhas and Vukina (1999) or Levy and Vukina (2004), we fix the output margin by assuming common mortality rate and the target weight of finished birds. This approach significantly simplifies the problem because, assuming fixed and common  $y_i$  reduces the payment mechanism in (2) from a piece-rate tournament to a standard cardinal tournament, where  $b$  is no longer a base piece rate but rather a simple salary.<sup>5</sup> This way, the actual production contract is reduced into a contest of who can produce the target output with the lowest cost (feed utilization).<sup>6</sup>

Formally, let's consider an  $N$ -player cardinal tournament game in which  $N$  risk-averse growers contract with a risk-neutral integrator the production of broiler chickens. Each grower  $i$  ( $i = 1, 2, \dots, N$ ) aims to produce the same amount of chickens, normalized to 1 pound, with a combination of inputs (chicks, feed, medication, etc.) which is assumed to be feasible in the sense of correctly reflecting target weight of finished broilers and nutritionally meaningful feed-conversion ratio.<sup>7</sup> The performance of grower  $i$  is specified as

$$x_i = \underline{x} + \frac{\bar{x} - \underline{x}}{1 + e_i u_i}, \quad (3)$$

where  $x_i = \frac{c_i}{y_i}$  and  $y_i = 1$  for all  $i$  so  $x_i$  measures the settlement cost per one pound of chickens produced,  $e_i$  is effort and  $u_i$  is the productivity shock for grower  $i$ . This specification implies that if the grower exerts 0 effort, then the settlement cost will be  $\bar{x}$ , which represents the upper bound determined by prevailing technology (nutrition, genetics, housing design, etc.).

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<sup>5</sup>Under these assumptions, Tsoulouhas and Vukina (1999) have shown that the tournament contract used by the poultry industry is in fact the first-order approximation of an optimal contract.

<sup>6</sup>Vukina and Zheng (2009) use an alternative specification by normalizing the feed margin. They show that empirically the two specifications fit the data equally well.

<sup>7</sup>We implicitly assume constant returns to scale production technology and therefore this normalization is innocuous.

By exerting effort, the grower can decrease the settlement cost, which also depends on the productivity shock  $u_i$ . In the limit, when the grower exerts large effort and the productivity shock is very high, the grower can reduce the settlement cost to  $\underline{x}$ , the lower bound determined by the current technology.<sup>8</sup>

The stochastic nature of the production technology is characterized by the productivity shock. We define the productivity shock in a broad sense. It includes factors that are specific to each individual grower, like the equipment failure, illness in the family, etc., as well as factors that are common to all growers in the same tournament, such as outside temperature, humidity, feed formula, etc. Therefore, we assume that the productivity shocks to different growers in the same tournament, that is,  $(u_1, u_2, \dots, u_N)$ , have the joint distribution of  $G(\cdot)$ . Each grower only learns  $u_i$  after the production process is complete but it is common knowledge that the productivity shocks are drawn from the density.

Based on the adopted simplifications, the grower payment (2) can be written as

$$w_i = b + \beta \left( \frac{1}{N} \sum_j x_j - x_i \right),$$

and her utility function is specified to be CARA,

$$U(w_i, e_i) = -\exp[-r(w_i - C(e_i))]$$

where  $w_i$  denotes the total revenue,  $C(e_i)$  denotes the cost of effort and  $r$  is the growers' risk aversion parameter. All standard assumptions regarding the cost function apply, that is,  $C' > 0$  and  $C'' > 0$ . For simplicity, we assume  $C(e_i) = \frac{\gamma}{2}e_i^2$  with  $\gamma > 0$ .

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<sup>8</sup>Animal husbandry is characterized by animals eating *ad libidum* (at will), that is, the feed is always there for them to eat. So, even if the grower does absolutely nothing, the birds will still eat and grow, although the total settlement cost will be higher relative to the situation where the grower did everything possible to create the chicken house environment conducive to efficient metabolism.

### 3.1 Characterization of the Equilibrium

When growers make decisions on how much effort to exert, the productivity shocks  $u_i$  ( $i = 1, \dots, N$ ) have not yet been realized. Therefore, in this tournament game, ex ante, growers are the same and have the same information regarding other structural elements of the game. In such a case, a symmetric equilibrium is a natural outcome to analyze. The optimal strategy  $e^*$  is based on each grower's maximizing her ex-ante expected payoff with respect to  $e_i$ . After integrating out all the unknowns and assuming that all other growers adopt the same strategy, the expected payoff function for grower  $i$  can be written as

$$\max_{e_i} EU_i = \int U(w_i, e_i) dG(u_1, u_2, \dots, u_N).$$

The first order condition for the maximization problem can be written

$$\begin{aligned} & \gamma e_i \int \exp(-r\pi_i) dG(u_1, u_2, \dots, u_N) \\ &= \beta \frac{N-1}{N} (\bar{x} - \underline{x}) \int \frac{u_i}{(1+e_i u_i)^2} \exp(-r\pi_i) dG(u_1, u_2, \dots, u_N), \end{aligned}$$

where  $\pi_i = w_i - C(e_i)$ . Furthermore, the second order sufficient condition is

$$r \int \exp(-r\pi_i) \left\{ \begin{array}{l} -2\beta \frac{N-1}{N} (\bar{x} - \underline{x}) \frac{u_i^2}{(1+e_i u_i)^3} \\ -\gamma - r \left[ \beta \frac{N-1}{N} (\bar{x} - \underline{x}) \frac{u_i}{(1+e_i u_i)^2} - \gamma e_i \right]^2 \end{array} \right\} dG(u_1, u_2, \dots, u_N),$$

which is negative. Hence, a solution to this maximization problem exists and is unique.

Since each agent is the same ex ante, in equilibrium, each grower exerts the same effort  $e^*$ , which is implicitly given by

$$\begin{aligned} & \gamma e^* \int \exp(-r\pi_i^*) dG(u_1, u_2, \dots, u_N) \\ &= \beta \frac{N-1}{N} (\bar{x} - \underline{x}) \int \frac{u_i}{(1+e^* u_i)^2} \exp(-r\pi_i^*) dG(u_1, u_2, \dots, u_N), \end{aligned} \tag{4}$$

where

$$\pi_i^* = b + \beta \frac{1-N}{N} \frac{(\bar{x} - \underline{x})}{1+e^* u_i} + \frac{\beta}{N} (\bar{x} - \underline{x}) \sum_{j \neq i} \frac{1}{1+e^* u_j} - \frac{\gamma}{2} e^{*2}.$$

## 4 Structural Estimation

As explained in detail in Section 2, our data set is an unbalanced panel where  $\bar{N}$  growers that grow chickens for the same integrator compete in different tournaments of size  $N < \bar{N}$ . Denoting  $x_{it}$  as the grower  $i$ 's ( $i = 1, \dots, \bar{N}$ ) performance in tournament  $t$  ( $t = 1, \dots, T$ ), we can rewrite (3) as

$$\tilde{x}_{it} = \log \left( \frac{\bar{x} - x_{it}}{x_{it} - \underline{x}} \right) = \log e_t^* + \tilde{u}_{it}, \quad (5)$$

where  $\tilde{u}_{it} = \log(u_{it})$ . We further specify the marginal distribution for  $\tilde{u}_{it}$  as normal with mean  $\mu_u$  and variance  $\sigma_u^2$ . As a result, the marginal distribution for  $\tilde{x}_{it}$  conditional on  $\log e_t^*$  is normal with mean  $\log e_t^* + \mu_u$  and variance  $\sigma_u^2$ . To model the joint distribution of  $(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t})$  conditional on  $\log e_t^*$ , we use the copula approach (Nelsen, 1999).

Specifically, by Sklar's theorem (Sklar (1973)), for a joint distribution  $H(\tilde{x}_1, \dots, \tilde{x}_N)$ , there is a unique copula  $C$ , such that  $C(H_1(\tilde{x}_1), \dots, H_N(\tilde{x}_N)) = H(\tilde{x}_1, \dots, \tilde{x}_N)$ . One important class of copulas, namely, the Archimedean copulas, has been extensively studied in the statistics literature and has found wide applications in modeling positive dependence in empirical work. For an Archimedean copula, the copula  $C$  can be expressed as  $C(u_1, \dots, u_n) = \varphi^{[-1]}(\varphi(u_1) + \dots + \varphi(u_n))$ , where  $\varphi$  is a generator of the copula and is a decreasing and convex function, and  $\varphi^{[-1]}$  denotes the pseudo-inverse of  $\varphi$ . The family of Archimedean copulas include a wide range of copulas. For example, the generator  $\varphi(u) = \frac{1}{q}(u^{-q} - 1)$  corresponds to the widely used Clayton copula.

We assume that the joint distribution of the productivity shocks  $(\tilde{u}_{1t}, \dots, \tilde{u}_{N_t t})$  can be modeled by a Clayton copula.<sup>9</sup> Then the joint distribution of  $(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t})$  conditional on  $\log e_t^*$  can be written as

$$H(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t}) = \left[ \sum_{i=1}^{N_t} \Phi \left( \frac{\tilde{x}_{it} - \log e_t^* - \mu_u}{\sigma_u} \right)^{-q} - N_t + 1 \right]^{-1/q},$$

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<sup>9</sup>For a Clayton copula, it can be verified that it satisfies the conditions for an affiliated distribution given in Milgrom and Weber (1982). For the discussion of the connection between the concept of affiliation (positive dependence) and copulas see Hubbard, Li and Paarsch (2009) and the references therein.

where  $\Phi(\cdot)$  is the CDF for standard normal and  $q > 0$ . The conditional joint density for  $(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t})$  is then

$$\begin{aligned} h(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t}) &= \frac{\partial^{N_t} H(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t})}{\partial \tilde{x}_{1t} \dots \partial \tilde{x}_{N_t t}} \\ &= q^{N_t - 1} \left(\frac{1}{q} + 1\right) \dots \left(\frac{1}{q} + N_t - 1\right) \left[ \sum_{i=1}^{N_t} \Phi\left(\frac{\tilde{x}_{it} - \log e_t^* - \mu_u}{\sigma_u}\right)^{-q} - N_t + 1 \right]^{-1/q - N_t} \\ &\quad \left[ \prod_{i=1}^{N_t} \Phi\left(\frac{\tilde{x}_{it} - \log e_t^* - \mu_u}{\sigma_u}\right) \right]^{-q-1} \prod_{i=1}^{N_t} \left[ \frac{1}{\sigma_u} \phi\left(\frac{\tilde{x}_{it} - \log e_t^* - \mu_u}{\sigma_u}\right) \right] \end{aligned} \quad (6)$$

where  $\phi(\cdot)$  is the pdf for standard normal. From (4), it is clear that  $e_t^*$  is a nonlinear function of all of the structural parameters:  $q$ ,  $\gamma$ ,  $\mu_u$ ,  $r$  and  $\sigma_u^2$ . These are the parameters to be estimated.

In principle, the structural parameters of the model can be estimated using the maximum likelihood estimation (MLE) method based on (6). However, we encountered numerical difficulties when applying this approach. These difficulties arise because of the following two facts. First, the likelihood (6) is a fairly complex nonlinear function of the underlying structural parameters to be estimated. Second, at each iteration of the maximization procedure (i.e. for each trial vector of the parameter values), we need to solve for the equilibrium effort level  $e_t^*$  using (4) for each tournament. In other words, the implementation of the MLE involves a nested algorithm to calculate the optimal effort level. Since (4) involves a high-dimensional ( $N_t$  dimensional) integral, we need to use simulation methods to numerically solve for the equilibrium effort level. This further complicates the computation.

Therefore, exploiting certain features of the structural model, we propose the following multi-step computationally easy approach to estimate the model. In the first step, we conduct the nonlinear least squares (NLS) in (5) using the pooled data.<sup>10</sup> In principle, the NLS yields consistent estimates for all the parameters. However, because of the highly nonlinear nature of the optimal effort level  $e_t^*$ , the NLS could behave poorly in finite samples. Nevertheless, the NLS estimates can be used as initial estimates for some iterative procedure to obtain

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<sup>10</sup>In fact, the NLS estimation here also involves a nested algorithm in calculating the optimal effort level implicitly defined in (4).

more stable estimates for the entire set of structural parameters. In our case, we have found through experimentation that the NLS produces sensible estimates for all parameters but the copula dependence parameter  $q$ . To obtain a sensible estimate for  $q$ , we note that from the assumptions we have made,  $(\tilde{u}_{1t}, \dots, \tilde{u}_{N_t t})$  has the joint density,

$$h(\tilde{u}_{1t}, \dots, \tilde{u}_{N_t t}) = q^{N_t - 1} \left(\frac{1}{q} + 1\right) \dots \left(\frac{1}{q} + N_t - 1\right) \left[ \sum_{i=1}^{N_t} \Phi\left(\frac{\tilde{u}_{it} - \mu_u}{\sigma_u}\right)^{-q} - N_t + 1 \right]^{-1/q - N_t} \left[ \prod_{i=1}^{N_t} \Phi\left(\frac{\tilde{u}_{it} - \mu_u}{\sigma_u}\right) \right]^{-q-1} \prod_{i=1}^{N_t} \left[ \frac{1}{\sigma_u} \phi\left(\frac{\tilde{u}_{it} - \mu_u}{\sigma_u}\right) \right], \quad (7)$$

which only depends on three parameters,  $q$ ,  $\mu_u$ , and  $\sigma_u^2$ . Therefore, in the second step, we use the NLS residuals  $\hat{\varepsilon}_{it}$  recovered from the first step to maximize the likelihood function based on (7) to obtain  $\hat{q}$  with  $\mu_u$  and  $\sigma_u^2$  replaced by  $\hat{\mu}_u$  and  $\hat{\sigma}_u^2$  obtained in the first step.

## 4.1 Estimation Results

Estimation results for the copula specification presented above are collected in Table 1. During estimation, when we use (4) to numerically solve for the equilibrium effort, we use importance sampling with 1,000 simulations to evaluate the integrals in the equation.

Now all the structural parameters have been recovered. If we further assume the participation constraint for the agent is binding on average, we can use the structural parameters to compute the reservation utility  $\underline{U}$ , that is,

$$\begin{aligned} \underline{U} &= \frac{1}{T} \sum_{t=1}^T EU_{it}(e_t^*) \\ &= -\frac{1}{T} \sum_{t=1}^T \int \exp \left\{ -r \left[ b + \beta \frac{1-N}{N} \frac{(\bar{x} - \underline{x})}{1 + e_t^* u_{it}} + \frac{\beta}{N} (\bar{x} - \underline{x}) \sum_{j \neq i} \frac{1}{1 + e_t^* u_{jt}} - \frac{\gamma}{2} e_t^{*2} \right] \right\} \\ &\quad dG(u_{1t}, u_{2t}, \dots, u_{N_t t}). \end{aligned} \quad (8)$$

The multi-dimensional integral in (8) will be evaluated using simulations. In more detail, we simulate  $(u_{1t}, u_{2t}, \dots, u_{N_t t})$  from  $G(u_{1t}, u_{2t}, \dots, u_{N_t t})$ . The procedure for simulating  $(u_{1t}, u_{2t}, \dots, u_{N_t t})$  from  $G(u_{1t}, u_{2t}, \dots, u_{N_t t})$  is based on Wu, Valdez and Sherris (2006: pp. 6-7) as follows:

1. Generate  $N_t$  independent uniform (0,1) variables. Denote them by  $w_{1t}, \dots, w_{N_t t}$ .

2. For  $k = 1, \dots, N_t - 1$ , set  $s_{kt} = w_{kt}^{1/k}$ .
3. Set  $t = F_T^{-1}(w_{N_t t})$  where

$$F_T(t) = t + \sum_{k=1}^{N_t-1} \frac{1}{k!} t^{(1+k\rho)} \prod_{j=0}^{k-1} (1 + j\rho) \left[ \frac{t^{-\rho} - 1}{\rho} \right]^k.$$

4. Set

$$\begin{aligned} m_{1t} &= \left[ 1 + s_{1t} \dots s_{(N_t-1)t} (t^{-\rho} - 1) \right]^{-1/\rho} \\ m_{N_t t} &= \left[ 1 + (1 - s_{(N_t-1)t}) (t^{-\rho} - 1) \right]^{-1/\rho} \\ m_{kt} &= \left[ 1 + (1 - s_{(k-1)t}) \prod_{j=k}^{N_t-1} s_j (t^{-\rho} - 1) \right]^{-1/\rho} \quad \text{for } k = 2, \dots, N_t - 1. \end{aligned}$$

5. The desired values are

$$u_{kt} = \Phi^{-1}(m_{kt}) \quad \text{for any } k$$

where  $\Phi$  is the cdf for normal with mean  $\mu_u$  and variance  $\sigma_u^2$ .

We obtain an estimate for  $\underline{U} = -0.9855$ .

## 4.2 Model Selection

With the linear additive specification (1) frequently used in the literature and the assumption that  $\tilde{v}_{it}$  ( $= \log(v_{it})$ ) is normally distributed with mean  $\mu_v$  and variance  $\sigma_v^2$ , the conditional joint *pdf* for performance measure  $\tilde{x}_i$  in (6) becomes

$$h(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t} | \eta_t) = \prod_{i=1}^{N_t} \left[ \frac{1}{\sigma_v} \phi \left( \frac{\tilde{x}_{it} - \log e_t^* - \tilde{\eta}_t - \mu_v}{\sigma_v} \right) \right]. \quad (9)$$

If we further specify the common shock  $\tilde{\eta}_t$  ( $= \log(\eta_t)$ ) to be normally distributed with mean  $\mu_\eta$  and variance  $\sigma_\eta^2$ , then (9) becomes

$$h(\tilde{x}_{1t}, \dots, \tilde{x}_{N_t t}) = \int \prod_{i=1}^{N_t} \left[ \frac{1}{\sigma_v} \phi \left( \frac{\tilde{x}_{it} - \log e_t^* - \tilde{\eta}_t - \mu_v}{\sigma_v} \right) \right] d\Phi \left[ \frac{\tilde{\eta}_t - \mu_\eta}{\sigma_\eta} \right]. \quad (10)$$

For identification purpose,  $\mu_\eta$  is normalized to be 0 and only  $\mu_v$  is estimated. As the likelihood function from the model with the standard specification is much simpler than that of the copula specification, we use MLE to estimate all the structural parameters in one step. Estimation results for this specification are collected in Table 2. Using the estimates for the structural parameters, we obtain an estimate for  $\underline{U} = -0.9999$ .

With two sets of parameter estimates from the two specifications, a natural question to ask is which specification fits the data better. Since we estimate the two specifications using different econometric approaches, we use the mean squared prediction error to evaluate which specification fits the data better. More specifically, in our context, the mean squared prediction error is defined as

$$MSPE = \frac{1}{M} \sum_{it=1}^M (\tilde{x}_{it} - \hat{\tilde{x}}_{it})^2,$$

where  $M$  is the total number of observations in the dataset and  $\hat{\tilde{x}}_{it}$  is the predicted value of  $\tilde{x}_{it}$  from the structural model. With the copula specification,  $\hat{\tilde{x}}_{it} = \log \hat{e}_t^* + \hat{\mu}_v$ . With the standard specification,  $\hat{\tilde{x}}_{it} = \log \hat{e}_t^* + \hat{\mu}_v$ . Results show that the MSPEs are 0.1809 and 0.1863 for the copula specification and the standard specification, respectively. This indicates that the copula specification fits the data slightly better.

## 5 Welfare Simulations

The structural estimates we obtain enable us to conduct various counterfactual simulations. The process involves two steps. First, we need to obtain the estimates of the optimal contract parameters by using the estimates of the structural parameters of the two models (copula

specification and standard specification). These estimated optimal contract parameters can then be used to quantify welfare measurements of economic interest.

We are interested in two sets of results. First we want to compare the performance of the standard linear additive cardinal tournament model with the tournament model where the shocks are modeled using the copula approach. Second, we want to compare the performance of the cardinal tournament model with a much simpler piece rates model under both standard and copula approach.

As discussed above, tournament is the dominant payment mechanism in the modern broiler industry. However, many other production contracts used in the very similar industries, such as swine and turkeys, use mechanisms known as fixed performance standards (Thsoulouhas and Vukina 1999; Levy and Vukina 2004). Under such schemes, growers are paid a base payment corrected by a bonus payment calculated by comparing some individual performance measure (e.g., feed conversion) against a predetermined fixed standard. Under the same set assumptions that we used to convert the actual payment mechanism used by the broiler industry (2) to the standard cardinal tournament, the fixed performance standards can be easily converted into simple piece rates. The only important difference between a tournament and a fixed standard lies in the computation of the benchmark against which the performance of an individual grower is compared. Whereas in case of tournaments, benchmark is determined by the contest among growers, in the other case, it represents a predetermined technological constant.

Based on the existing literature on welfare comparison of tournaments and piece rates (Levy and Vukina 2004), it follows that the agents' welfare under two schemes depends on the magnitude of common production shock relative to the sum of the variance of growers's ability and the variance of idiosyncratic shock. The results show that if the variance of common shocks exceeded the sum of the variance of grower ability and the variance of the idiosyncratic shock, the payments to growers for a single tournament will have less variance than under a simple piece rate. The same result holds for the sequence of tournaments over any time horizon when tournament groups are drawn randomly. Because in our case

growers are assumed to be ex-ante identical, the variance of their ability is zero, and the welfare comparison of two schemes should depend on the relative magnitudes of common shock versus idiosyncratic shock. When common shock dominates, tournaments are welfare superior to piece rates and when idiosyncratic shock dominates, piece rates are better.

The simulation results that follow are based on three critical assumptions that are implicit in everything that has been done so far, but for clarity, need to be re-iterated. First, we assume the monopolistic principal (integrator) such that the optimal matching between multiple principals and agents is not an issue. Secondly, the problem is even further simplified by assuming that all growers are ex ante identical so their participation constraints should be binding and they should *ex-ante* collect zero rents. These two assumptions collectively generate the situation where measuring the aggregate welfare of the principal-agent relationship reduces to measuring principal's profits. Finally, because of the fact that in our model we normalized the output margin to unity (1 pound) and assumed the total number of growers under contract to be exogenous, the principal's profit maximization problem becomes a cost minimization problem.

## 5.1 Optimal Tournament Contract Parameters

In a tournament with multiple growers, the principal's total cost consists of the cost of producing 1 pound of live chicken meat (settlement cost) per grower plus the grower compensation, everything summed over all growers:

$$\begin{aligned}
TC_{Rt}(e_i^* \forall i) &= \sum_{i=1}^{N_t} (x_{it} + w_{it}) \\
&= N_t (b_t + \underline{x}) + \left[ 1 - \beta_t \frac{(N_t - 1)}{N_t} \right] (\bar{x} - \underline{x}) \sum_{i=1}^{N_t} \frac{1}{1 + e_i^* u_{it}} \\
&\quad + \beta_t \frac{(\bar{x} - \underline{x})}{N_t} \sum_{i=1}^{N_t} \sum_{j \neq i} \frac{1}{1 + e_i^* u_{jt}}.
\end{aligned}$$

As is standard in the contract literature, the principal is assumed to be risk neutral. Therefore, the expected total cost is

$$\begin{aligned}
ETC_{Rt} &= \int \left\{ \left[ 1 - \beta_t \frac{(N_t-1)}{N_t} \right] (\bar{x} - \underline{x}) \sum_{i=1}^{N_t} \frac{1}{1+e_t^* u_{it}} + \beta_t \frac{(\bar{x}-\underline{x})}{N_t} \sum_{i=1}^{N_t} \sum_{j \neq i} \frac{1}{1+e_t^* u_{jt}} \right\} dG(u_{1t}, u_{2t}, \dots, u_{Nt}) \\
&\quad + N_t (b_t + \underline{x}) \\
&= N_t (\bar{x} - \underline{x}) \int \frac{1}{1 + e_t^* u_{it}} dG_i(u_{it}) + N_t (b_t + \underline{x}), \tag{11}
\end{aligned}$$

where  $G_i(u_{it})$  is the marginal CDF for  $u_{it}$ . Here, the contract parameters  $b$  and  $\beta$  vary with  $t$  because the number of players  $N_t$  in each tournament is different, leading to a different set of optimal contract parameters for each tournament. Because in the observed data, the contract parameters are the same in all tournaments, in our empirical estimation, they do not vary over  $t$  either.<sup>11</sup>

The optimal contract parameters are obtained as follows. First, using the earlier mentioned assumption that the agent's participation constraint is binding on average<sup>12</sup> that is,  $EU_{it}(e_t^*) = \underline{U}$  (from (8) above without  $\frac{1}{T} \sum_{t=1}^T$ ),  $b_t$  can be written as a function of  $\beta_t$  and other structural parameters including  $G(\cdot)$ . Then, we find the first order condition for the minimization of  $ETC_{Rt}$  with respect to  $\beta_t$  to get the  $\beta_t^{opt}$ , which in turn can be used to obtain  $b_t^{opt}$  and  $ETC_{Rt}^{opt}$ .

## 5.2 Optimal Piece Rate Contract Parameters

When the piece rate contract is used, the payment to grower  $i$  can be written as

$$w_i = b_p - \beta_p x_i,$$

and her utility function becomes

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<sup>11</sup>There are many different reasons for why the observed contracts usually depart from the theoretically optimal contracts. The legal and bureaucratic costs of writing a different contract for each new batch of birds could be prohibitively high and could effectively wipe out all gains associated with constant fine tuning of contract parameters.

<sup>12</sup>In fact, since in this case all growers are assumed to be ex-ante identical, the participation constraints should be binding at the individual grower's level as well.

$$\begin{aligned}
U(w_i, e) &= -\exp\left\{-r\left[b_p - \beta_p x_i - \frac{\gamma}{2}e^2\right]\right\} \\
&= -\exp\left\{-r\left[b_p - \beta_p\left(\underline{x} + \frac{\bar{x} - \underline{x}}{1 + eu_i}\right) - \frac{\gamma}{2}e^2\right]\right\}.
\end{aligned}$$

As a result, the agent's maximization problem is

$$\max_e EU_i = \int U(w_i, e) dG_i(u_i),$$

and the optimal solution for the effort is characterized by the following FOC,

$$\int \exp(-rw_i^*) \left[ \beta_p (\bar{x} - \underline{x}) \frac{u_i}{(1 + e^*u_i)^2} - \gamma e^* \right] dG_i(u_i) = 0,$$

where  $w_i^* = b_p - \beta_p \left(\underline{x} + \frac{\bar{x} - \underline{x}}{1 + e^*u_i}\right)$ .

Under the piece rate, the principal's total cost is

$$\begin{aligned}
TC_{pt}(e_t^*) &= \sum_{i=1}^{N_t} (x_{it} + w_{it}) \\
&= \sum_{i=1}^{N_t} \left[ b_{pt} + (1 - \beta_{pt}) \left( \underline{x} + \frac{\bar{x} - \underline{x}}{1 + e_t^* u_{it}} \right) \right] \\
&= N_t b_{pt} + N_t (1 - \beta_{pt}) \underline{x} + (1 - \beta_{pt}) (\bar{x} - \underline{x}) \sum_{i=1}^{N_t} \frac{1}{1 + e_t^* u_{it}}.
\end{aligned}$$

Therefore, the expected total cost is

$$\begin{aligned}
ETC_{pt} &= N_t (1 - \beta_{pt}) (\bar{x} - \underline{x}) \int \frac{1}{1 + e_t^* u_{it}} dG_i(u_{it}) \\
&\quad + N_t b_{pt} + N_t (1 - \beta_{pt}) \underline{x}.
\end{aligned}$$

The optimal contract parameters are obtained in a similar fashion as for the tournament contract. First, using the assumption that the agent's participation constraint is binding on average, that is,  $EU_{it}(e_t^*) = \underline{U}$ ,  $b_{pt}$  can be written as a function of  $\beta_{pt}$  and other structural

parameters including  $G(\cdot)$ . In piece rate,  $EU_{it}(e_t^*)$  becomes

$$- \int \exp \left\{ -r \left[ b_{pt} - \beta_{pt} \left( \underline{x} + \frac{\bar{x} - \underline{x}}{1 + e_t^* u_{it}} \right) - \frac{\gamma}{2} e_t^{*2} \right] \right\} dG_i(u_{it}).$$

Then, we use the first order condition of  $ETC_{pt}$  with respect to  $\beta_{pt}$  to get the  $\beta_{pt}^{opt}$ , which in turn leads to  $b_{pt}^{opt}$  and  $ETC_{pt}^{opt}$ .

### 5.3 Simulation Results

First, the estimates of the optimal tournament contract parameters can be used to quantify the principal's average expected total cost either per tournament or per grower. For instance, we find that the average expected total cost per tournament using optimal contract parameters under the affiliation specification is \$9.5768, while the average expected total cost per tournament using optimal contract parameters under the standard (linear additive) specification is \$8.8255. Recall, that the reason for why these numbers are so small is because in our model we normalized the quantity of output to be only 1 pound and the tournament competition was about who can produce this target level of output with smallest possible cost. Comparing these results we see that the standard specification of production shocks tends to underestimate the principal's average expected total cost by about 7.85% relative to the affiliation approach.<sup>13</sup>

If, for illustration purposes, we take the actual number of pounds of live chickens produced under those contracts and multiply them with the above results, we find that with our copula (affiliation) specification, the principal's average expected total cost per tournament using the optimal contract parameters is \$2,302,167, and the average expected total cost per grower across tournaments is \$73,742. On the other hand, under the standard specification, the principals average expected total cost per tournament and per grower across tournaments using the optimal contract parameters are \$2,121,561 and \$67,957, respectively. These numbers clearly show the significant sensitivity of the cost estimates to production shocks

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<sup>13</sup>As a reference, the average actual (observed) settlement cost in the data is \$9.7588 per tournament.

modeling specifications.

The second analysis we conduct is to study whether the tournament mechanism or the piece rate payment mechanism is better for the principal, under the condition that optimal contract parameters are used. We conduct the analysis using the set of structural parameter estimates from the copula specification. We find that when the optimal piece rate contract is used, the average expected total cost per tournament (group) is \$9.5272, which is lower than the average expected total cost per tournament when the optimal tournament contract is used. This indicates that in our context, the variance in output due to the common shock is not large enough relative to the variance due to the individual shocks to make the use of tournament as a more profitable compensation scheme for the principal. Furthermore, when  $q$  is set to be 10 times of the current estimate, that is, at the value of 9.251, the use of optimal tournament contracts yields an average expected total cost per tournament \$11.3511, which is lower than \$11.3915, the average expected total cost per tournament when the optimal piece rate contract is used. This result is consistent with the theory as higher affiliation means the shocks for different growers are more positively dependent on one another. As a result, the variance in output due to the common shock is larger than the variance due to the individual shocks.

## 6 Conclusions

In this paper we estimate an empirical cardinal tournament model in which the players' productivity shocks are positively dependent. We propose to use the affiliation approach to model the positive dependence of the productivity shocks. Our specification is more general than the standard linear and additive specification used in the literature to date. We estimated our model using the settlement data from the contracts that govern the production of broiler chickens. The econometric results show that affiliation approach to modeling the production shocks fits the data better than the standard linear additive model in terms of the selection criterion such as the mean square prediction error. Two approaches also yield

quite different estimates of the average expected total cost of contracting and consequently would also give different estimates of integrators profits and agents total compensations. In situations where those cannot be perfectly observed and the policy recommendations or court decisions need to rely on econometric estimates, this example clearly shows the importance of modeling assumptions.

The results obtained in this paper also raise another very interesting issue that has been frequently mentioned in the empirical contract literature but never resolved in a satisfactory manner. The issue is about almost universally detected discrepancy between the theoretically optimal contracts and those actually observed in real life. The analysis of the contract settlement data coming from confined animal feeding operations, such as contracts for production of broiler chickens, provides an ideal environment for the reality checks of various theoretical models and estimation procedures. What exactly is at stake is easily understood by looking at some fixed genetic and nutritional constraints governing the production of broiler chickens. Recall that the average feed conversion ratio in our data set is 2.03, that is, it takes 2.03 pounds of chicken feed to produce 1 pound of live weight gain. The worst feed conversion recorded in this data is 3.38 and the best (minimum) feed conversion is 1.83.

Assume now that the empirically observed contract is suboptimal relative to its theoretically correct counterpart. Without getting into explaining the reasons why this suboptimal contract exists, let just assume that we want to replace the observed contract with the theoretically optimal contract to measure the impact of this management innovation via an improved contract design that would more efficiently solve asymmetric information problems (either moral hazard or adverse selection or both). Notice that if this contract substitution produces the improvement in feed conversion ratio for chickens for more than about 11%, which is the difference between the average recorded feed conversion and the best recorded feed conversion, the result is automatically highly suspicious because it claims that some organizational and management innovation can accomplish something that the decades-long research in nutrition and genetics was not able to do.

Back to our results. The obtained cost difference between two approaches to model

production shocks is 7.85% and the difference between the optimal parameters affiliation specification and the actual cost is 1.86% and the difference between the optimal parameters standard specification and the actual cost is 9.56%. Given the fact that the cost of producing chickens are largely determined by the feed conversion ratio, one can see that the numbers are well inside the meaningful range of experimentation with the contract design. The whole issue requires further research into this subject matter. We believe that this paper is a step in the right direction.

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**Table 1:** Estimation Results from the Copula Specification

	Estimate	Stan. Err.	<i>t</i> -stat
$q$	0.9251	0.0228	40.65
$\gamma$	0.3976	0.0524	7.58
$\mu_a$	2.9686	0.0636	46.69
$r$	0.5667	0.0787	7.20
$\sigma_u^2$	0.0956	0.0246	3.89

**Table 2:** Estimation Results for the Standard Specification

	Estimate	Stan. Err.	<i>t</i> -stat
$\gamma$	0.4053	0.0019	211.06
$\mu_v$	3.0638	0.0148	206.98
$r$	0.0001	0.0005	0.18
$\sigma_v^2$	0.0164	0.0012	13.85
$\sigma_\eta^2$	0.2212	0.0027	82.50
log likelihood	4812.48		