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Tomislav Vukina^a & Xiaoyong Zheng^a

^a Department of Agricultural and Resource Economics, North Carolina State University, Campus Box 8109, Raleigh, NC 27695-8109

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Homogenous and Heterogenous Contestants in Piece Rate Tournaments: Theory and Empirical Analysis

Tomislav VUKINA and Xiaoyong ZHENG

Department of Agricultural and Resource Economics, North Carolina State University, Campus Box 8109, Raleigh, NC 27695-8109 (vukina@ncsu.edu; xzheng@ncsu.edu)

In this article we show that sorting different ability contestants in piece rate tournaments into more homogenous groups alters agents' incentives to exert effort. We propose a method for structurally estimating the piece rate tournament game with heterogenous players and apply it to the payroll data from a broiler production contract. Our counterfactual analysis shows that under reasonable assumptions, both the principal and the growers can gain when the tournament groups are heterogenized. This business strategy could be difficult to implement in real-life settings, however. This article has supplementary material online.

KEY WORDS: Heterogenous contestants; Piece rate tournament; Structural estimation.

1. INTRODUCTION

Tournaments are labor contracts in which an individual's payoff depends on his or her own performance relative to that of others. There are *rank-order* (*ordinal tournaments*), such as those considered by Lazear and Rosen (1981), where an individual player's payment depends on his or her rank within the group, and *cardinal tournaments*, where the reward is a continuous function (typically linear) of the difference between an individual player's performance and the group average performance. Cardinal tournaments are also referred to as *relative performance* compensation schemes (Nalebuff and Stiglitz 1983) or *yardstick competition* (Shleifer 1985). Yet another type of tournament frequently used in settlements of some production contracts (e.g., broiler chickens) is the *piece rate tournament* (Tsoulouhas and Vukina 1999). Piece rate tournaments are in fact variable piece rate schemes in which an individual player's piece rate varies in proportion with his or her performance relative to the group average performance.

Virtually all real-world tournaments are contests among players with unequal abilities. When players have different abilities, rank-order tournaments are known to exhibit some undesirable properties. For example, asymmetries in the knowledge of abilities entail inefficiencies, because contestants do not correctly self-sort into the leagues commensurate with their types. Correcting this problem may result in entry credentials and bigger prize spreads in leagues that target players of higher ability. With full knowledge of players' abilities, rank-order tournaments with players of heterogeneous abilities still suffer from incentive problems, because both high-ability and low-ability contestants tend to work less than their respective efficient effort levels. Handicapping and prize structures indexed by ability are required to correct for these inefficiencies (see McLaughlin 1988).

As for cardinal tournaments, the literature reports that they exhibit no efficiency losses associated with mixing players of uneven abilities. When rewards are linearly related to performance, better players have no incentive to stop exerting effort once they realize that they are going to win, and worse players

have no incentive to surrender once they realize that they are going to lose. Consequently, in cardinal tournaments organizers have no incentives to sort players into more homogenous groups, because the incremental reward for improved performance (penalty for worse performance) at the margin is the same whether a player is more or less able (Knoeber and Thurman 1994).

The welfare effects of mixing players of varying abilities in piece rate tournaments has not yet been reported. The best-known real business world example of a piece rate tournament is the settlement of the production contracts for broiler chickens. Tournaments are also used to settle contracts for the production of turkeys. Another type of variable piece rate, sometimes known as a fixed performance standard, where an individual agent's performance is compared against some fixed technological standard (e.g., a quality attribute) to determine the piece rate, are widely used in the contract production of wine grapes, various fruits and vegetables, eggs, swine, and other commodities (see Levy and Vukina 2004). Our motivation comes from observing that unlike in many sport competitions, in which organizers homogenize the contestants by placing them into similar ability leagues or divisions to enhance competition, poultry companies generally do not attempt to place their agents into more homogenous groups in which they compete for cost-efficiency bonuses. This fact is even more puzzling given the knowledge that via repeated contracting with the same pool of agents for a long period, company production managers can precisely discern the agents' abilities, yet do not exploit this information to their advantage.

The main tenet of this article is that making groups of contestants in piece rate tournaments more homogenous or more heterogenous creates different incentives for agents to exert effort. Under certain assumptions, we show that for a given mean of the tournament group's heterogeneity parameters, larger variance (i.e., more heterogenous agents) induces higher optimal

effort. Thus the principal always wins by mixing contestants of different abilities rather than sorting them into more homogenous groups. The effect on the growers is indeterminate, however, because higher optimal effort leads to both higher payment and higher cost of effort, making this an empirical question.

In addition to its theoretical contribution, this article also contributes to the growing literature on the structural econometrics approach to estimating tournament models, which proves to be quite useful for conducting counterfactual (policy) analyses. Several authors have estimated structural models of various types of tournaments. Ferrall and Smith (1999) estimated a sequential tournament game for championship series in sports. Ferrall (1996) and Chen and Shum (2010) estimated elimination tournament models for workers competing for limited promotion slots. Zheng and Vukina (2007) estimated a rank-order tournament model to quantify the efficiency gains of an organizational innovation that would replace an ordinal tournament with a cardinal tournament. More generally, our work is also related to the literature on estimating games of incomplete information and auction games in particular (e.g., Laffont, Ossard, and Vuong 1995; Hong and Shum 2002, 2003; Haile, Hong, and Shum 2003).

This article represents the first attempt to structurally estimate a piece rate tournament model that captures the most important features of the production contracts observed in the broiler chickens industry. Using broiler production contracts settlement data, we empirically quantified the effects of heterogenizing tournament groups for both the company and the contract growers. Our counterfactual analysis shows that under reasonable assumptions, both the principal and the growers gain when the tournament groups are heterogenized. This indicates that such a business strategy may be efficient. However, our results also show that such a practice would be very difficult to implement, because increasing group heterogeneity would almost invariably generate two countervailing effects that could erode its benefits.

2. INDUSTRY AND DATA

The poultry industry is often considered a role model for the industrialization of agriculture. The industry is entirely vertically integrated, from breeding flocks and hatcheries to feed mills, transportation divisions, and processing plants. The final (finishing) stage of production, when 1-day-old chicks are brought to the farm and then grown to market-weight broilers, is organized almost entirely through contracts between integrators and independent growers. Large national companies, including Tyson Foods, Pilgrim's Pride, and Perdue Farms, dominate broiler contract production. These companies run their operations through smaller divisions spread throughout the country, but mainly in the southeast.

Modern broiler production contracts are agreements between an integrator company and growers that bind farmers to tend the company's chickens until they reach market weight by strictly following specific production practices in exchange for monetary compensation. According to a typical contract, the grower provides land, housing facilities, utilities (electricity and water)

and labor and pays for operating expenses, such as repairs and maintenance, cleanup, and manure and mortality disposal. The company provides chicks, feed, medication, and the services of field men. Most modern broiler contracts are settled using a two-part piece-rate tournament. In this type of tournament, the total payment R_i to grower i is the sum of the base rate and the bonus rate multiplied by the live pounds of poultry moved from the grower's farm,

$$R_i = \left[a + b \left(\frac{1}{N} \sum_{j=1}^N \frac{c_j}{y_j} - \frac{c_i}{y_i} \right) \right] y_i. \quad (1)$$

As seen from (1), individual grower's piece rate per pound of live poultry produced is the sum of a constant base rate a (e.g., 3.5–4.5 cents per pound) and a variable bonus rate determined by the grower's relative performance. The bonus rate is determined as a percentage, b , of the difference between group average performance, $\frac{1}{N} \sum_{j=1}^N \frac{c_j}{y_j}$, and the producer's individual performance, $f_i = \frac{c_i}{y_i}$. Settlement costs, c_i , are obtained by adding chicks, feed, medication, and other customary flock costs and then divided this sum by the number of pounds of live chickens produced. Calculation of the group average performance includes all N growers whose flocks are harvested at the same time. For all growers to be exposed to the same common production shock, tournaments are usually settled once a week. For the below-average settlement cost per pound of chicken produced (above-average performance), the grower receives a bonus, and for the above-average settlement cost per pound of chicken produced, he or she receives a penalty.

As explained by Tsoulouhas and Vukina (1999), poultry tournaments are double-margin contests about who can produce more output (live poultry) with the smallest possible settlement cost. The growers' effort (husbandry practices) stochastically influences the settlement costs (feed utilization) and the quantity of output. Growers can economize with feed (and hence settlement costs, c_i) by preventing spillage through proper maintenance of feeders and storage bins and by maintaining a housing environment that is conducive to efficient feed conversion. Growers also can separately influence output (live poultry weight, y_i) by undertaking actions aimed at preventing excessive animal mortality. Depending on the size of the chickens grown, the grow-out process takes about 7–8 weeks.

An interesting feature of these contracts is that the composition of the tournaments (i.e., settlement groups) is governed predominantly by timing and logistics of the production process and not with an attempt to form more homogenous or more diverse groups of contestants. From the perspective of an individual grower, participation in a given tournament is entirely exogenous. The process can be described as follows. Every time a new batch of chicks needs to be housed, the production manager looks at his list of growers with currently empty houses. The decision on who gets a new batch is based primarily on how many days/weeks have elapsed since the last flock was harvested from a farm. The companies try to be fair by giving the same number of flocks per year to all growers, because more flocks mean larger profits due to significant fixed costs. Other factors possibly influencing the decision could be the location (with the objective of minimizing the transportation cost

of hauling feed and birds to the farms) and biosecurity reasons (i.e., the need for some farms to be temporarily vacant due to a disease). Growers complain about unpredictably long downtime periods (which companies use as a supply-response device), and it is highly unlikely that a grower would refuse the delivery of a new batch for strategic reasons, because he or she might end up waiting for quite some time before the delivery of another batch is scheduled. Also, given the fact that the contracts are short-term, the receipt of a new batch automatically renews the old expired contract, whereas the refusal of a new batch without a solid verifiable reason could be construed as a unilateral contract termination. The composition of the tournament groups is determined by the timing of harvesting. Typically, the tournament group consists of all growers whose birds were harvested within the same calendar week. The better the grower, the faster his or her birds will reach market weight and the faster his or her house will open up again for the acceptance of a new batch.

The data set used in this study comprises broiler production information gathered from the payroll data of one company's profit center whose production contract corresponds to the payment scheme described in (1). Each observation in the data set represents one contract settlement, that is, the payment received and the grower's performance associated with one grower and one flock of birds delivered to the integrator's processing plant. The data, obtained from "settlement sheets," contain information on the quantities and costs of various inputs supplied by the integrator (e.g., chicks, feed, medication, vaccinations), the number of birds placed and harvested, the quantity of broiler meat (live weight) produced, production start and termination dates, mortality rates, and other factors.

The settlement dates range from July 1995 to July 1997, totaling 104 tournaments (one tournament per week). The total number of growers is 356, and the total number of usable observations is 3247 flocks. The average live weight of the fully grown broilers is 4.81 pounds (maximum, 5.75 pounds; minimum, 3.88 pounds), the average period to grow chickens to that weight is 53 days (maximum, 79 days; minimum, 43 days), and the average feed conversion ratio (i.e., pounds of feed necessary to produce one pound of live animal weight gain) is 2.03 (maximum, 3.38; minimum, 1.83). The variable piece rate ranges from 2.4 cents to 5.3 cents per pound.

Participation ranges between 1 and 12 times (average, 9.12 times). The mode of the distribution is 11, with 147 growers participating in 11 tournaments. Given the fact that maximum participation is 12, then the participation rate can be calculated as $3247/(12 * 356)$, or 76%. Another way to view the participation rate is to look at the durations of the downtime periods (i.e., the number of days between the settlement of a previous tournament and the start of the next one). This gap can be as short as 3 days and as long as 109 days. On average, this gap is 16.2 days (median, 12 days). An obvious reason why some growers participated in fewer tournaments is because they joined the profit center or left the business during the period covered by our data set. For cases that do not fit this explanation, the differences in participation rates can be explained by the differences in times that individual growers

need to grow birds to market weight and to clean and prepare the chicken houses for new flocks, and also by the integrator's idiosyncrasies in scheduling the delivery of new chicks.

3. PIECE RATE TOURNAMENT MODEL

The exact modeling of a tournament game that would simultaneously take into account both the feed margin and the output margin obviously is quite complex, if not impossible, in terms of both theoretical modeling and econometric estimation. The literature on broiler tournaments (e.g., Knoeber and Thurman 1994; Tsoulouhas and Vukina 1999; Levy and Vukina 2004) fixes the output margin by assuming a common mortality rate and target weight of finished animals. This approach significantly simplifies the problem, because assuming fixed and common y_i reduces the payment mechanism in (1) from a piece rate tournament to a standard cardinal tournament, in which a is no longer a base piece rate but now is a simple salary. In this way, the actual production contract is reduced into a contest of who can produce the target output with the lowest cost (feed utilization). Because the received theory predicts no effects associated with mixing contestants of different abilities in standard cardinal tournaments, this model specification trivializes our problem. Integrators do not organize growers into more homogeneous groups, because this practice has no effect on growers' performance or on the integrator's profits.

In an alternative specification, a double-margin tournament contract can be simplified by fixing the settlement cost margin. Under this assumption, the actual tournament becomes a contest about who can produce more output (live weight) with a fixed amount of inputs. Consider a N -player piece rate tournament game in which N risk-neutral growers contract with a risk-neutral integrator for the production of broiler chickens. Each grower, i ($i = 1, 2, \dots, N$), is given the same combination of inputs (e.g., chicks, feed, medication) worth c_i dollars, normalized to \$1. Given these inputs, the performance of grower i is specified as

$$f_i = \frac{c_i}{y_i} = \frac{1}{y_i} = \frac{1}{\theta_i e_i u_i \eta}, \quad (2)$$

where y_i indicates the pounds of live poultry produced, e_i is grower i 's effort, and θ_i is the grower's idiosyncratic ability (efficiency) parameter. We broadly define grower ability as inherent or acquired skills resulting from experience, education, age, as well as other grower-specific factors, such as location, quality, and vintage of the production facilities and equipment. Higher θ_i implies that a grower can combine inputs and effort more efficiently in the production of broiler meat. We assume that from grower i 's perspective, θ_j , $\forall j \neq i$, the abilities of other growers in the same tournament, are random variables drawn from a distribution $G(\cdot)$ with lower bound $\underline{\theta} \geq 0$. Distribution $G(\cdot)$ is twice continuously differentiable and has density $g(\cdot)$ that is strictly positive on the support. This specification captures the real-life situation in which growers typically do not know their opponents in a particular tournament, but do know the distribution of other growers' abilities through repeated participation in similar tournaments over an extended period.

The stochastic production technology is characterized by two types of shocks. Both grower i 's idiosyncratic productiv-

ity shock u_i (e.g., equipment failure, sick child) and the common productivity shock η (e.g., outside temperature, humidity, feed formula) materialize slowly during the production process. Shocks u_i and η are assumed to be drawn from distributions $F(\cdot)$ with lower bound $\underline{u} \geq 0$ and $P(\cdot)$ with lower bound $\underline{\eta} \geq 0$, respectively. Both $F(\cdot)$ and $P(\cdot)$ are twice continuously differentiable and have densities $f(\cdot)$ and $p(\cdot)$ that are strictly positive on the support. Each grower learns the values of u_i and η only after the production process is complete, but it is common knowledge that the two shocks are drawn from the two densities. Finally, we assume that θ_i , u_i , and η are independent of one another.

The grower payment can be written as

$$R_i = \left[a + b \left(\frac{1}{N} \sum_j f_j - f_i \right) \right] \frac{1}{f_i},$$

and his or her payoff function is given by $\pi_i = R_i - C(e_i)$, where R_i denotes the total revenue and $C(e_i)$ denotes the cost of effort. All standard assumptions regarding the cost function apply, that is, $C' > 0$ and $C'' > 0$. In particular, we assume that $C(e_i) = \frac{\gamma}{2} e_i^2$ with $\gamma > 0$ such that the model has a closed-form solution. The quadratic cost function specification is without loss of generality, in the sense that the model would be observationally equivalent for any pair of productivity function, $\theta_i e_i^x u_i \eta$, and cost function, $\frac{\gamma}{2} e_i^{2x}$, where $x > 0$.

3.1 Characterization of the Equilibrium

When growers make decisions on how much effort to exert, the idiosyncratic productivity shocks, u_i ($i = 1, \dots, N$), and the common productivity shock, η , have not yet been realized. Thus in this tournament game, ex ante, growers differ only in terms of their own ability and have the same information regarding other structural elements of the game. In such a case, symmetric equilibrium is a natural outcome to analyze. The optimal strategy, $e_i^* = s(\theta_i)$, is based on each grower's maximizing his or her ex ante expected payoff with respect to e_i . After integrating out all of the unknowns and assuming that all other growers adopt the same strategy, $e_j^* = s(\theta_j)$, for $j \neq i$, the expected payoff function for grower i can be written as

$$\begin{aligned} E\pi_i &= \int \dots \int (R_i - C(e_i)) \\ &\quad \times \prod_{j \neq i} g(\theta_j) \prod_{i=1}^N f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^N du_i d\eta \\ &= \int \dots \int \left\{ a\theta_i e_i u_i \eta + b \frac{1-N}{N} + b \frac{1}{N} \sum_{j \neq i} \frac{\theta_i e_i u_i}{\theta_j e_j u_j} - C(e_i) \right\} \\ &\quad \times \prod_{j \neq i} g(\theta_j) \prod_{i=1}^N f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^N du_i d\eta. \end{aligned} \tag{3}$$

Now we are in the position to state the following result.

Proposition 1. The unique symmetric pure-strategy Bayesian Nash equilibrium, $e_i^* = s(\theta_i)$ ($i = 1, \dots, N$), of this piece rate

tournament game is

$$\begin{aligned} e_i^* &= s(\theta_i) \\ &= \frac{\theta_i a E(\eta) E(u)}{2\gamma} \\ &\quad + \theta_i \sqrt{\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N-1)}{N\gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)}, \end{aligned} \tag{4}$$

where $E(\cdot)$ denotes the mean of the random variable in parentheses.

Proof. See the online Supplement.

3.2 Comparative Statistics

With the closed-form solution for optimal effort e_i^* , it is easy to study various comparative statics results. First, optimal effort, e_i^* , is increasing in ability θ_i , the base rate a , the slope of the bonus piece rate b , and the expectation of the common shock $E(\eta)$, and decreasing in the marginal cost of effort γ . On the other hand, the comparative static result with respect to $E(u)$ is ambiguous, because $E\left(\frac{1}{u}\right)$ is a nonlinear function of $E(u)$.

Second, notice that

$$E\left(\frac{1}{\theta^2}\right) = E^2\left(\frac{1}{\theta}\right) + V\left(\frac{1}{\theta}\right),$$

where $V(\cdot)$ denotes the variance of the random variable in parentheses. Thus, for a constant $E\left(\frac{1}{\theta}\right)$, increasing $V\left(\frac{1}{\theta}\right)$ increases the optimal effort e_i^* . Because θ is defined as the growers' ability (efficiency) parameter, $\frac{1}{\theta}$ can be considered an inaptitude parameter. This implies that for a given mean of the growers' inaptitude parameters, larger variance (i.e., more heterogenous growers) produces higher optimal effort. This means that any grower, i , given his or her own ability, will exert more effort when competing against a highly diversified group of growers than when competing in a more homogenous group of contestants. The intuition for this result is as follows. If two players in the tournament are replaced by two new players, one with extremely high ability and one with extremely low ability, without a change in the average grower ability in the tournament group, then the increase in the expected group average performance due to the presence of a grower with extremely high ability outweighs the decrease in the expected group average performance due to the presence of a grower with extremely low ability, resulting in an overall increase in expected group average performance. This is because a grower's expected performance is increasing in ability at an increasing rate, as can be best seen from (8) below. Given a grower's own expected performance, the higher the expected group average performance, the higher the incentives to exert effort either to close the gap (if his or her expected performance is below the expected group average performance) or widen the gap (if his or her expected performance is higher than the expected group average performance), because both situations lead to higher payments.

Finally, we also note that the optimal effort is increasing in the number of tournament contestants N . In our variable piece rate scheme, the magnitude of the piece rate depends on the difference between the group average performance and a

grower's own performance. The larger the difference, the higher the payment, and thus the greater the incentive to exert effort. With small number of contestants, a grower's own performance can more readily influence the group average performance, and thus the incentive to exert high effort is dampened because a grower's own high effort increases the benchmark for comparison. On the other hand, in groups with a large number of contestants, a grower's performance will not influence the group average performance significantly, and thus exerting high effort would, ceteris paribus, produce larger increase in the difference between a grower's performance and the group average performance.

3.3 The Principal's Problem

We now turn to the principal's side of the model. We follow the literature on empirical testing of contract theories when contracts are taken as given, and model the behavior of the principal under the observed contractual terms without assuming an optimal contract design. Works on this paradigm include those of Paarsch and Shearer (2000), Ferrall and Shearer (1999), and Shearer (2004), among others. In an actual business environment, many factors could prevent the principal from using efficient contracts. For example, Ferrall and Shearer (1999) estimated that the transaction cost that prevents a firm from adopting the optimal contract can be as large as the cost associated with incomplete information in a principal agent model.

We propose a simplified version of the principal's optimization problem that is consistent with the observed contract form. First, we assume that the principal uses piece rate tournament contracts to maximize the long-term profitability of its individual profit centers. Second, we assume that the profit center manager has perfect foresight about the future downstream demand for processed chicken meat and can correctly plan ahead for the logistics of production and processing over the entire sample period. In practical terms, this means that the decisions regarding the number of contestants in each tournament and the group composition of each tournament have been made for all of the tournaments at the beginning of the sample period and are taken as given. Third, at the beginning of the sample period, each grower knows the number of tournaments in which he or she will participate and the number of contestants that he or she will face in each tournament. Finally, we assume that growers have unlimited access to credit markets to smooth their current consumption patterns, and that their reservation utilities are exogenous and normalized to 0. Given these assumptions, the principal's optimization problem is reduced to simply finding a pair of contract parameters, a and b , such that its total long-term expected profit is maximized, subject to the growers' incentive compatibility and participation constraints.

Formally, the principal's total profit function over the sample period can be written as

$$P = \sum_{t=1}^T \sum_{i=1}^{N_t} \left\{ \left[p - a - b \left(\frac{1}{N_t} \sum_{j=1}^{N_t} f_{jt} - f_{it} \right) \right] \frac{1}{f_{it}} - c_{it} \right\},$$

where p is the market price of processed chicken meat (net of processing cost), c_{it} is the unit production cost normalized to \$1, the discount factor is assumed to be 1, T is the total number

of tournaments in the data set, and N_t is the number of contestants in tournament t . The total expected payoff can then be written as

$$EP = \sum_{t=1}^T \sum_{i=1}^{N_t} \left\{ (p - a) \theta_i e_{it} E(u) E(\eta) + \frac{(N_t - 1)}{N_t} b - b \frac{1}{N_t} \sum_{j \neq i} \frac{\theta_i e_{it}}{\theta_j e_{jt}} E(u) E\left(\frac{1}{u}\right) - 1 \right\}, \quad (5)$$

and the principal's optimization problem becomes

$$\max_{a,b} EP(a, b, e_{it}) \quad \text{s.t.} \quad e_{it} = e_{it}^*, \quad \forall i, t \quad \text{and}$$

$$\sum_{t=1}^{T_i} E\pi_{it}(e_{it}^*) \geq 0; \quad \forall i.$$

The first constraint is the incentive compatibility constraint; the second set of constraints is the set of participation constraints, one for each grower; and T_i is the number of tournaments in which grower i participates during the sample period. The participation constraints reflect the assumption about unlimited access to credit and the ability to smooth consumption such that all growers care about is that, on average, their expected reservation utilities are covered.

After incorporating the incentive constraint into the profit maximization by replacing the effort level by its optimal value and by invoking the assumption of zero reservation utilities for all agents, the foregoing problem can be written as

$$\max_{a,b} EP(a, b, e_{it}^*) \quad \text{s.t.} \quad \sum_{t=1}^{T_i} E\pi_{it}(e_{it}^*) \geq 0.$$

The newly introduced notation \underline{i} denotes the grower with the lowest ability in tournament t , and $T_{\underline{i}}$ is the number of tournaments in which this grower participates during the sample period. Notice that the number of participation constraints has been conveniently reduced from one per grower to only one. This is possible because from (3) and (4), it follows that

$$\begin{aligned} E\pi_{it}(e_{it}^*) &= \theta_i^2 \left[\frac{a^2 E^2(\eta) E^2(u)}{4\gamma} + \frac{a E(u) E(\eta)}{2} \right. \\ &\quad \times \sqrt{\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)} \\ &\quad \left. + \frac{b(N_t - 1)}{2N_t} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right) \right] + \frac{b(1 - N_t)}{N_t} \end{aligned}$$

is monotonically increasing in θ_i . Thus in any tournament, as long as the expected payoff is greater or equal to zero for the grower with the lowest ability, the payoff is automatically greater than zero for all other growers in the same tournament. The reduction of the number of participation constraints also requires the assumption that all other growers participate in the same set of tournaments as the grower with the lowest ability. However, in the data, different growers participate in different sets of tournaments. We ignore this discrepancy between theory and empirical analysis for tractability reasons.

The Lagrangian function of the modified constrained maximization problem can be written as

$$L = EP(a, b, e_{it}^*) + \lambda \sum_{i=1}^{T_i} E\pi_{it}(e_{it}^*).$$

There is no way to determine whether or not the participation constraint for the agent with the lowest ability is binding. There are two possible solutions to this problem. If the participation constraint is not binding, then the optimal solution is characterized by the following two first-order conditions:

$$\frac{\partial EP(a, b, e_{it}^*)}{\partial a} = 0, \quad \frac{\partial EP(a, b, e_{it}^*)}{\partial b} = 0. \quad (6)$$

Note that e_{it}^* is also a function of a and b . In contrast, if the participation constraint is binding, then the optimal solution is characterized by the following two equations:

$$\begin{aligned} \sum_{i=1}^{T_i} E\pi_{it}(e_{it}^*) &= 0, \\ \frac{\partial EP(a, b, e_{it}^* \forall i, t)}{\partial a} \frac{\partial \sum_{i=1}^{T_i} E\pi_{it}(e_{it}^*)}{\partial b} &= \frac{\partial EP(a, b, e_{it}^* \forall i, t)}{\partial b} \frac{\partial \sum_{i=1}^{T_i} E\pi_{it}(e_{it}^*)}{\partial a}. \end{aligned} \quad (7)$$

The analytical details for the foregoing results are available in the online Supplement. We denote the case where the participation constrained is not binding by M1 and the case where the participation constrained is binding by M2. In the empirical section, we estimate both models and use a model selection technique to select between the two models.

4. STRUCTURAL ESTIMATION

Here we estimate the model using the structural approach. There are two reasons for this. First, the main goal of our empirical analysis is to identify the effect of group heterogeneity on contestants' efforts and outputs. If data from several companies (or several regions) with sufficient variation in $E(\frac{1}{\theta^2})$ or $V(\frac{1}{\theta})$ were available, then we could use reduced-form regression to test whether agents' efforts, and thus their outputs, depend on the heterogeneity of the tournament groups in the way predicted by our model. Given that our data come from one regional profit center of one company, the reduced-form econometrics approach is not applicable, because we have only one observation for $E(\frac{1}{\theta^2})$. The structural approach, on the other hand, imposes restrictions on the data through the model specification such that the desired effect can be identified. A caveat of the structural econometrics approach is that the results obtained depend on these functional form assumptions. However, our assumptions are fairly standard in the literature, and, more importantly, some are actually testable. Second, along with testing model implications, we are also interested in quantifying the effects of changing the heterogeneity of the tournament groups on both the contestants and the principal. This is only possible when a structural model is estimated.

4.1 Estimation Strategy

As explained in detail in Section 2, our data set is an unbalanced panel in which \bar{N} growers that grow chickens for the same integrator compete in different tournaments of size $N < \bar{N}$. Denoting f_{it} as the observation in tournament t ($t = 1, \dots, T$) that records grower i 's ($i = 1, \dots, \bar{N}$) performance, we can rewrite (2) as

$$\begin{aligned} \frac{1}{f_{it}} &= \theta_i e_{it} u_{it} \eta_t \\ &= \theta_i^2 \left[\frac{aE(\eta)E(u)}{2\gamma} \right. \\ &\quad \left. + \sqrt{\frac{a^2 E^2(\eta)E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u)E\left(\frac{1}{u}\right)E\left(\frac{1}{\theta^2}\right)} \right] \\ &\quad \times u_{it} \eta_t, \end{aligned} \quad (8)$$

where the second equality follows from (4). Taking logarithms of both sides of (8) yields

$$\begin{aligned} \log \frac{1}{f_{it}} &= 2 \log \theta_i + \log \left[\frac{aE(\eta)E(u)}{2\gamma} \right. \\ &\quad \left. + \sqrt{\frac{a^2 E^2(\eta)E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u)E\left(\frac{1}{u}\right)E\left(\frac{1}{\theta^2}\right)} \right] \\ &\quad + \log u_{it} + \log \eta_t \\ &= 2 \log \theta_i + \log z_t + \log \eta_t + \log u_{it}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} z_t &= \left[\frac{aE(\eta)E(u)}{2\gamma} \right. \\ &\quad \left. + \sqrt{\frac{a^2 E^2(\eta)E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u)E\left(\frac{1}{u}\right)E\left(\frac{1}{\theta^2}\right)} \right], \end{aligned}$$

varies only across tournaments because of its dependence on N_t . If N_t is fixed across tournaments, then z_t is also fixed across tournaments, because other terms, such as $E(u)$, $E(\frac{1}{u})$, $E(\frac{1}{\theta^2})$, and $E(\eta)$, are all fixed constants. For ease of presentation, we use the following notation: $E(\log u_{it}) = m_u$, $\log \theta_1 = m_\theta$ where θ_1 is the ability of the first grower and $E(\log \eta_t) = m_\eta$.

The estimation strategy has two stages. In the first stage, we propose the following ordinary least squares (OLS) regression:

$$\log \frac{1}{f_{it}} = \delta_0 + \sum_{i=2}^{\bar{N}} \mu_i d_{it} + \sum_{t=2}^T \lambda_t g_{it} + \epsilon_{it}, \quad (10)$$

where δ_0 is a constant. To avoid multicollinearity, we excluded the dummy variable for the first grower and the dummy variable for the first tournament. Thus the coefficient on grower i 's dummy, $\hat{\mu}_i$ ($d_{it} = 1$, if the observation records grower i 's performance in tournament t , $d_{it} = 0$ elsewhere), is used to obtain an estimate of grower i 's ability from (9) relative to the excluded first grower, that is, $\hat{\mu}_i = 2 \log \theta_i - 2 \log \theta_1$. Similarly, the coefficient $\hat{\lambda}_t$ associated with the t th tournament dummy g_{it} is used to estimate the sum of the deterministic part of the output function (9) that varies only across tournaments and the common shock, all relative to the first (excluded) tournament, that is $\hat{\lambda}_t = (\log z_t + \log \eta_t) - (\log z_1 + \log \eta_1)$. As a result, the

constant $\widehat{\delta}_0$ is an estimate of $2 \log \theta_1 + (\log z_1 + \log \eta_1) + m_u$, or $\widehat{\delta}_0 = 2m_\theta + (\log z_1 + \log \eta_1) + m_u$ using the new notation. Given these results, grower i 's ability can be estimated using $\widehat{\theta}_i = \exp(\frac{\widehat{\mu}_i + 2m_\theta}{2})$. In addition, the estimated residual term $\widehat{\epsilon}_{it}$ can be used to estimate the log of the idiosyncratic productivity shock because $\log u_{it} = \epsilon_{it} + m_u$, and thus $\widehat{u}_{it} = \exp(\widehat{\epsilon}_{it} + m_u)$. Furthermore, using these estimates, the unknown parameters, $E(u)$, $E(\frac{1}{u})$, and $E(\frac{1}{\theta^2})$, can be easily constructed as follows:

$$\begin{aligned}\widehat{E}(u) &= \frac{1}{T} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \exp(\widehat{\epsilon}_{it} + m_u), \\ \widehat{E}\left(\frac{1}{u}\right) &= \frac{1}{T} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \exp(-\widehat{\epsilon}_{it} - m_u), \\ \widehat{E}\left(\frac{1}{\theta^2}\right) &= \frac{1}{N} \sum_{i=1}^{\bar{N}} \exp(-\widehat{\mu}_i - 2m_\theta).\end{aligned}\quad (11)$$

The fact that our unbalanced panel contains very few observations for some contestants can lead to biased estimates of μ_i for these contestants, and thus a biased estimate of $\widehat{E}(\frac{1}{\theta^2})$. The bias for μ_i is $\frac{1}{T_i} \sum_{t=1}^{T_i} \epsilon_{it}$, and the order of magnitude of this bias is $\frac{\sigma_\epsilon}{\sqrt{T_i}}$, where σ_ϵ is the standard deviation for ϵ_{it} . Because the bias is smaller for growers with more observations, the effect is mitigated by using the weighted average formula (as suggested by an associate editor),

$$\widehat{E}\left(\frac{1}{\theta^2}\right) = \frac{1}{N} \sum_{i=1}^{\bar{N}} w_i \exp(-\widehat{\mu}_i - 2m_\theta), \quad (12)$$

where $w_i = \exp(-\frac{\sigma_\epsilon}{\sqrt{T_i}}) / \sum_{i=1}^{\bar{N}} \exp(-\frac{\sigma_\epsilon}{\sqrt{T_i}})$. By design, observations from growers with more observations are given more weight.

In the second stage, we exploit the following relationship:

$$\begin{aligned}\lambda_t &= \log z_t + \log \eta_t - (\log z_1 + \log \eta_1) \\ &= \log z_t + \log \eta_t - (\delta_0 - 2m_\theta - m_u).\end{aligned}\quad (13)$$

From the first stage, we obtain an estimate for λ_t . We assume that $\log \eta_t$ is normally distributed with mean m_η and variance σ_η^2 . This assumption, although strong, is necessary for overidentification purposes. Without this assumption, the number of moments used in estimation would increase along with the number of unknown parameters. As a result, the number of moment conditions would always equal the number of unknown parameters to be estimated, and the generalized method-of-moments (GMM) overidentification test could not be performed. This assumption leads to $E(\eta) = \exp(m_\eta + \frac{\sigma_\eta^2}{2})$. As a result, the only unknowns in (13) are m_u , m_θ , m_η , γ , and σ_η^2 .

We estimate the unknown parameters with the GMM test by matching five population moments implied by the model with the sample moments of our data. The first three moments are $E(\lambda_t)$, $E(\exp(\lambda_t))$, and $E(\exp(2\lambda_t))$, and the last two moments are from the equations characterizing the principal's optimal solution, that is, (6) for model M1 and (7) for model M2. The last two moments are obtained by taking expectations on both sides of (6). We use the consistent but inefficient GMM estimation procedure, in which the identity matrix is used as the

weighting matrix. The two-step optimal GMM estimation is also feasible but not preferred. In such a procedure, the optimal weighting matrix is the inverse of the variance-covariance matrix of the sample moments evaluated at the first-step estimates. In our case, the first three moments have T observations, one for each tournament; however, there is only one observation for the last two moments. Thus, to obtain the variance-covariance matrix of the sample moments, we need to simulate the model using first-step estimates $(T - 1)$ times such that we can have T observations for the last two moments as well. Because we are already using the bootstrap procedure to obtain the correct standard errors for the second-stage estimates, using the two-step optimal GMM estimation procedure for the second-stage estimation will make the computation burdensome. Given that this stage of estimation procedure uses variables generated from the results of the first stage, we will obtain standard errors of the second stage using the bootstrap method. This completes the structural estimation of the model.

4.2 Identification

Before estimating the model, we need to determine whether all unknown parameters in the second stage of the estimation are identified. As shown in the online Supplement, the two parameters in the moment conditions m_η and m_u always appear together as a sum, and thus they cannot be identified separately. Only the sum can be identified. Identifying these parameters requires a normalization assumption. We normalize m_u to be 0 such that only four unknown parameters remain. Having five moments allows us to conduct an overidentification test.

4.3 Estimation Results

In the first stage, we run a simple OLS regression of (10) and obtain $R^2 = 0.9408$. The estimation results, together with the summary statistics for the dependent variable, $\log \frac{1}{f_{it}}$, are reported in Table 1.

Using the results from the first stage, we estimate $E(u)$, $E(\frac{1}{u})$, and $E(\frac{1}{\theta^2})$ using (11) and (12), and then proceed with the second-stage GMM estimation. In the data set, the base piece rate a is 3 cents per pound of live weight, and the marginal bonus rate b is 0.1. During the sample period, the average market price of processed chicken meat was about 59 cents a pound. From the data, we also can compute the average output per \$1 worth of input, which turns out to be 3.2341 pounds of live chicken, yielding a grow-out cost of about 31 cents per live pound. The average grower payment is 9.71 cents per \$1

Table 1. Estimates from the first-stage estimation

	Mean	Standard deviation	Number of observations
$\log \frac{1}{f_{it}}$	1.1683	0.1040	3247
$\widehat{\delta}_0$	1.2515		
$\widehat{\mu}_i, i \geq 2$	-0.0166	0.0186	355
$\widehat{\lambda}_t, t \geq 2$	-0.0666	0.0990	103
$\widehat{\epsilon}_{it}$	$-3.8352 * 10^{-13}$	0.0253	3247

Table 2. Estimates from the second-stage estimation

	M1			M2		
	Estimate	Std. err.	<i>t</i> -stat	Estimate	Std. err.	<i>t</i> -stat
m_θ	1.8769	0.0048	393.40	1.6917	0.0062	273.00
m_η	-3.1665	$1.9565 * 10^{-5}$	$-1.62 * 10^5$	-1.8114	0.0067	-269.93
σ_η^2	1.0175	$9.8207 * 10^{-6}$	$1.04 * 10^5$	0.0074	0.0035	2.10
γ	0.1172	$3.0214 * 10^{-5}$	$3.88 * 10^3$	0.5753	0.0053	107.78
<i>J</i> statistic	0.4320			$5.6507 * 10^{-4}$		

NOTE: Standard errors are based on 200 iterations of bootstrap.

worth of input, or about 3 cents per pound, as indicated by the base piece rate coefficient. Using a 70% yield to convert live pounds into meat pounds and then subtracting the feed costs and the grower payments from the market price leaves us with 49 cents per live pound to cover the processing, labor, overhead, and other principal’s costs, as well as some average rate of return. Given these simple back-of-the-envelope calculations, we choose the relevant price faced by the principal p as 0.50. Changing the value of p to 0.55 yields very similar results.

Estimation results for models M1 and M2 are collected in Table 2. Because the variance of the common shock, σ_η^2 , is highly significant in both cases, common shock is obviously important. This can be considered a specification test against the alternative specification that would exclude the common shock from the production function (8). In addition, the cost-of-effort parameter is positive in both cases, which is consistent with the model assumptions. In both models, the *J* statistics are <3.84, which is the critical value for χ^2 distribution with 1 degree of freedom and 5% significance level. These results indicate that for both models, we cannot reject the hypothesis that the model is correctly specified. Finally, for the M1 case, we have assumed that the participation constraint is not binding. Using the estimation results, we can evaluate whether or not this assumption holds. Our calculations show that the total profit that the grower with the lowest ability gets from all tournaments in which he or she participated is \$0.0194 per 1\$ worth of inputs, which is >0 and is consistent with the underlying assumption that the participation constraint is not binding.

To obtain the correct standard errors for the second-stage estimates and preserve the unbalanced panel structure of the data set, we use the nonparametric residual bootstrap method (e.g., Wooldridge 2002, p. 380). The exact procedure is as follows. From the first-stage estimation, we recover the residuals, $\hat{\epsilon}_{it}$, for each observation. Then, at each iteration of the bootstrap, we resample with replacement from the recovered residuals to obtain a new sample of residuals. We add back the new sample of residuals to $\hat{\delta}_0 + \sum_{i=2}^N \hat{\mu}_i d_{it} + \sum_{t=2}^T \hat{\lambda}_t g_{it}$ to obtain a new sample of $\log \frac{1}{f_{it}}$. Finally, we repeat the entire estimation (both the first-stage and second-stage) using the new sample of data. We repeat this procedure 200 times. The bootstrap standard deviation of the second-stage parameter estimates are used as the standard errors.

Finally, using the structural estimates, we are able to compute all quantities of economic interest for each observation. The results are collected in Table 3. For model M1, we find

that on average, a grower exerts 0.9696 unit of effort per \$1 worth of inputs, and the cost associated with this effort level is 5.51 cents. As a result, on average a grower earns 9.71 cents in total payment, and his or her profit amounts to 4.20 cents per \$1 worth of inputs. Results from model M2 are similar.

4.4 Model Selection

From Tables 2 and 3, we notice that the two models yield different sets of estimates for the structural parameters and the quantities of economic interest. Some differences are small, whereas others are big. A natural question to ask then is which model fits the data better. To answer this question, we use the in-sample mean squared error of prediction (MSEP) for λ_t , $\exp(\lambda_t)$, and $\exp(2\lambda_t)$, respectively, using the estimated structural parameters from the two models. A smaller value of MSEP indicates that the model fits the data better.

The results, given in Table 4, show that the difference in the MSEP for λ_t in the two models is insignificant. But the MSEPs for $\exp(\lambda_t)$ and $\exp(2\lambda_t)$ from M2 are significantly smaller than those from M1, indicating that M2 fits the data better. Standard errors for the difference in MSEP between the two models are obtained from the same bootstrap procedure for obtaining standard errors for the second-stage parameter estimates.

4.5 Specification Testing

As mentioned earlier, an alternative specification of broiler production contracts is obtained by assuming that the output level is constant across all growers. More specifically, assume that each grower has a contract to produce $y_i = 1$ pound of poultry, which makes this tournament a contest about who can produce a given output with smallest possible cost, c_i . Under these assumptions, the performance of grower i becomes

$$f_i = \frac{c_i}{y_i} = c_i, \tag{14}$$

Table 3. Quantities of economic interest

	M1		M2	
	Mean	Standard deviation	Mean	Standard deviation
R_{kit} (\$)	0.0971	0.0114	0.0971	0.0114
e_{it}	0.9696	0.0086	0.4343	0.0039
$C(e_{it})$ (\$)	0.0551	0.0010	0.0543	0.0010
$R_{kit} - C(e_{it})$ (\$)	0.0420	0.0112	0.0429	0.0112

Table 4. Selecting between M1 and M2

	M1 MSEP	M2 MSEP	(M1 MSEP – M2 MSEP) <i>t</i> -stat for the difference
λ_t	1.0150	1.0137	0.1698
$\exp(\lambda_t)$	0.2761	0.0130	42.52
$\exp(2\lambda_t)$	0.1609	$6.8247 * 10^{-4}$	47.49

NOTE: Standard errors are based on 200 iterations of bootstrap.

and the grower payment (1) reduces to a standard form of a cardinal tournament,

$$R_i = a + b \left(\frac{1}{N} \sum_j f_j - f_i \right).$$

A detailed exposition of the cardinal tournament model is presented in the online Supplement. Using the same set of specifications for f_i , $C(e_i)$, as well as the distributions of shocks and grower abilities, again the model yields two cases, which we denote by M3 and M4. Results from this specification are collected in Tables 5 and 6. We note that M3 produces unreasonably large and negative numbers for grower profits, which can be regarded as evidence against this model, although the model has passed the overidentification test. On the other hand, the results from M4 appear to be reasonable. Thus we conducted a model selection test between M4 and M2. The results, given in Table 7, show that for all three variables, the difference in MSEP from the two models is insignificant. This result indicates that we cannot empirically distinguish between the models, meaning that from a statistical standpoint, the piece rate tournament specification of the broiler production contracts is as good as the standard cardinal tournament specification.

5. COUNTERFACTUAL ANALYSIS

The most interesting theoretical result given in Section 3 shows that for a fixed $E(\frac{1}{\theta})$ (the mean of the growers' inaptitude parameters), an increase in its variance $V(\frac{1}{\theta})$, and thus in $E(\frac{1}{\theta^2})$, generates an increase in the equilibrium effort, e_i^* . This implies that the principal can gain from heterogenizing the tournament groups as long as he or she can sell the product at a price higher than the payment to the contract growers, and as long as he or she does not violate the growers' participation constraint. On the other hand, the effect of this policy change on growers' welfare is unclear. This is because higher effort leads to higher productivity and thus higher payment, but also

Table 6. Quantities of economic interest using the alternative specification

	M3		M4	
	Mean	Standard deviation	Mean	Standard deviation
R_{kit} (\$)	0.0971	0.0114	0.0971	0.0114
e_{it}	80.6386	0.2795	0.1887	0.0007
$C(e_{it})$ (\$)	$3.6187 * 10^4$	$0.0251 * 10^4$	0.0518	0.0004
$R_{kit} - C(e_{it})$ (\$)	$-3.6187 * 10^4$	$0.0251 * 10^4$	0.0454	0.0115

higher cost of effort. Another interesting result from the theoretical study is that effort increases as the number of contestants in a tournament increases. Because one way of heterogenizing the tournament groups is to decrease the tournament size (e.g., by excluding growers with medium abilities), examining the trade-off between more heterogenous contestants and smaller tournament size is of interest. One advantage of the structural econometrics approach is that it enables the use of model estimates to quantify the effects of such regime shifts or policy proposals.

To quantify these effects, we run a counterfactual experiment for a representative tournament. We choose the fourth tournament in our data set. As discussed in the previous section, based on the model selection test, M2 was preferred over M1. Although the parameter estimates from the two models were quite different, the quantities of economic interest obtained were similar. As it turns out, the results of the counterfactual analysis using estimates from these models are similar as well, and thus we present and discuss only the results based on M2.

5.1 Effects of Changing Group Heterogeneity

We first examine the effects of changing $E(\frac{1}{\theta^2})$. In the model, we assume that each grower in a tournament believes that his or her opponents are randomly drawn from the entire pool of 356 growers who work for the profit center. Thus the value of $E(\frac{1}{\theta^2})$ that the growers face can be estimated using (12). Using the structural estimates, this value is 0.0345. Assume that the principal announces that the growers who will participate in the coming tournament are to be drawn randomly from a group of growers with the value of $E(\frac{1}{\theta^2}) = \frac{1}{N_4} \sum_{i=1}^{N_4} w_i \frac{1}{\theta_i^2}$, where $N_4 = 42$ is the number of contestants in the fourth tournament. This number turns out to be 0.0346, which represents

Table 5. Estimates from the second-stage estimation using the alternative specification

	M3			M4		
	Estimate	Std. err.	<i>t</i> -stat	Estimate	Std. err.	<i>t</i> -stat
m_θ	7.9297	0.0066	$1.20 * 10^3$	1.2461	0.0537	23.20
m_η	-25.7378	0.0016	$-1.56 * 10^4$	0.3684	0.2063	1.79
σ_η^2	0.2256	0.0004	564.35	0.0237	0.0035	6.85
γ	11.1299	0.0001	$1.50 * 10^5$	2.9083	0.0366	79.47
<i>J</i> statistic	$9.1029 * 10^{-4}$			0.7054		

NOTE: Standard errors are based on 200 iterations of bootstrap.

Table 7. Selecting between M2 and M4

	M4 MSEP	M2 MSEP	(M4 MSEP – M2 MSEP) <i>t</i> -stat for the difference
λ_t	1.0109	1.0137	-0.19
$\exp(\lambda_t)$	0.0774	0.0130	0.88
$\exp(2\lambda_t)$	0.0244	$6.8247 * 10^{-4}$	0.07

NOTE: Standard errors are based on 200 iterations of bootstrap.

a 0.2075% increase in the value of $E(\frac{1}{\theta^2})$, meaning that the growers in this tournament are actually slightly more heterogenous than the pool of all 356 growers. Holding the productivity shocks at the level before the policy change, we can first compute the new optimal effort level for each grower, and then compute their outputs, payments, and profits under the new scenario. The results from such an experiment are collected in Table 8. We find that with the more heterogenous group, growers' equilibrium efforts (and thus their outputs and payments) would increase by about 0.10% on average. On the other hand, growers' costs of effort would increase by about 0.20% on average. However, because the absolute value of the revenue is larger than the cost of effort, growers' profits would increase by about 0.0087% on average. More specifically, the percent changes in growers' profits would range from 0.0046% to 0.0188%. These results clearly demonstrate that if the principal makes the tournaments more heterogenous, then the average contract grower effort, output, and payment would increase, and all growers would be better off than before the change.

It makes sense to translate the percentages into dollars. From Table 8, we see that for \$1 worth of inputs, a grower's output increases from 3.8158 pounds of chickens to 3.8195 pounds on average. Assuming that the industry is perfectly competitive, the principal can sell all of this extra output at the prevailing market price of 59 cents a pound. Thus, the principal receives a revenue of 0.22 cents by selling 0.0037 additional pounds of chicken meat. On the other hand, growers' profits increase from 6.0008 cents to 6.0013 cents on average, for a gain of 0.0005 cents. Consequently, heterogenizing the tournament group by 0.21% will result in a 0.2205-cent (per \$1 worth of inputs) increase in total gain by the principal and the growers.

Now suppose that this policy change can be applied to all growers in all tournaments, in which case the increase in social surplus would equal \$165.70 ($0.002205 \times \$75,145.91$) per grower. This is because the average settlement cost per grower in the data is 31.26 cents per pound and the average weight of poultry produced is 240,390 pounds per grower, implying an

average cost of inputs (feed, chicks, and other chargeable costs) of \$75,145.91 per grower. In reality, such a policy change is not possible, of course. Because a pool of growers under contract with a particular profit center is fixed, heterogenizing some tournament groups would require homogenizing other tournament groups. The effects from different tournaments would then cancel out each other. This is why we focus only on one tournament in our counterfactual analyses.

5.2 Trade-Off Between Tournament Size and Group Heterogeneity

We now examine the trade-off between tournament size and group heterogeneity. This counterfactual experiment is conducted as follows. Suppose that the principal announces that the second 21 growers in the fourth tournament will play in a tournament with a group size of 21, and that the participants of this new tournament will be randomly drawn from a group of growers with the value of $E(\frac{1}{\theta^2}) = \frac{1}{21} \sum_{i=1}^{21} w_i \frac{1}{\theta_i^2} = 3.4583 * 10^{-2}$, which represents a 0.0913% increase over $3.4551 * 10^{-2}$, the group heterogeneity measure for the fourth tournament as computed in the previous section. At the same time, this experiment also represents a 50% decrease in the tournament group size (from 42 to 21).

We examine the total surplus effect of changing these two factors simultaneously. Again the productivity shocks are held at the level before the policy change. Results are presented in Table 9. The reduction in group size has an opposite effect on effort, output, payments, and profits than that from heterogenizing the groups. Due to the large decrease (50%) in group size, this latter effect dominates the former effect, resulting in a 1.12% decrease in grower effort and output, a 1% decrease in payment, and a 2.22% decrease in the cost of grower effort on average. As a result, grower profits increase by 0.12% on average (range, 0.04%–0.19%).

6. CONCLUSIONS

In many sporting events, we observe the formation of leagues or divisions structured by the approximately even ability of teams or individual contestants. For example, in European football (soccer), England's Football Association divides clubs into different leagues depending on their strength. In the best league, the Premier League, 20 of the best teams, including Manchester United, Arsenal, and Liverpool, compete for the national title. At the end of each season, typically two teams exchange

Table 8. Effect of heterogenizing tournament groups

	Before		After		Change (%)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
$E(\frac{1}{\theta^2})$	$3.4480 * 10^{-2}$		$3.4551 * 10^{-2}$		0.2075	
$y_{kit} (\frac{1}{J_{kit}})$	3.8158	0.0843	3.8195	0.0844	0.0979	$1.8605 * 10^{-14}$
R_{kit} (\$)	0.1145	0.0047	0.1146	0.0047	0.0979	0.0019
e_{it}	0.3833	0.0034	0.3837	0.0034	0.0979	$7.8641 * 10^{-15}$
$C(e_{it})$ (\$)	0.0545	0.0010	0.0546	0.0010	0.1959	$1.8443 * 10^{-14}$
$R_{kit} - C(e_{it})$ (\$)	0.0600	0.0042	0.0600	0.0042	0.0087	0.0026

Table 9. Total effects of changing group heterogeneity and size simultaneously

	Before		After		Change (%)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
$E(\frac{1}{\theta^2})$	$3.4551 * 10^{-2}$		$3.4583 * 10^{-2}$		0.0913	
N	42		21		-50	
$y_{kit} (\frac{1}{f_{kit}})$	3.8136	0.0695	3.7711	0.0688	-1.1161	$1.4737 * 10^{-14}$
R_{kit} (\$)	0.1143	0.0039	0.1132	0.0039	-0.9962	0.0159
e_{it}	0.3835	0.0031	0.3792	0.0030	-1.1161	$8.3224 * 10^{-15}$
$C(e_{it})$ (\$)	0.0546	0.0009	0.0534	0.0008	-2.2179	$1.8877 * 10^{-14}$
$R_{kit} - C(e_{it})$ (\$)	0.0597	0.0035	0.0598	0.0035	0.1241	0.0381

leagues, such that the worst teams drop to the league immediately below and the best-placed teams from the league below advance to the league immediately above. The enhancement of competition is an intuitively obvious reason for such a prevalent practice. The motivation for this study came from observing that poultry integrators, unlike the Football Association, do not openly attempt to homogenize the settlement groups of contract growers for the purpose of creating more fierce competition among them. Instead, the composition of tournaments (settlement groups) seems to be governed by the timing and logistics of the production process, and the membership in those groups seem to change quite randomly.

Poultry production contracts are double-margin tournaments about who can produce more live poultry weight at the smallest possible cost. Because the exact modeling and structural estimation of these contests is extremely complex, if not impossible, the standard simplifying assumption in the literature is to fix the output margin. This assumption reduces the payment scheme in these contracts to a standard cardinal tournament where the competition is about who can produce the targeted output at the smallest possible cost. This approach trivializes our problem, because the equilibrium effort in this game ends up being independent of the variance of growers' abilities, and thus homogenizing tournament groups is irrelevant for efficiency.

In this article we propose an alternative specification in which instead of fixing the output margin, we fix the cost margin. This assumption gives rise to a piece rate tournament scheme (variable piece rate) where the competition is about who can produce more output at a given fixed cost. Our most interesting theoretical result shows that for a fixed mean of the growers' inaptitude parameters, an increase in its variance generates an increase in the equilibrium effort. This implies that the principal can actually gain from heterogenizing the tournament groups. On the other hand, the effect of this change on growers' welfare is unclear, because higher effort leads to higher productivity and thus higher payment, but also increases the cost of effort.

We estimated both models structurally and conducted the specification test, which indicated that we could not empirically distinguish between the two models. These results demonstrate that there is actually a very good reason why poultry integrators never attempt to homogenize the growers' settlement groups. Under the cardinal tournament specification, we obtain a trivial result that homogenizing groups would accomplish absolutely nothing. Under the piece rate tournament specification,

we obtain a somewhat unexpected result that heterogenizing the tournament groups would in fact benefit the integrator, whereas homogenizing them would hurt him, because the average equilibrium grower effort would decline and the production would suffer. Moreover, our counterfactual analysis shows that under reasonable assumptions, the growers also gain, indicating that such a business strategy may be efficient.

The foregoing result suggests that heterogenizing groups in piece rate tournaments may increase the integrator's profit. Now, instead of the original puzzle which we successfully solved, we are faced with a new puzzle: why poultry integrators do not try to assemble more heterogeneous groups of growers. Our counterfactual analyses indicate that such a practice would be very difficult to implement in practice. This is because increasing group heterogeneity will almost invariably generate two countervailing consequences that can erode its benefits. The first consequence is a decreased tournament group size; for example, one way to increase group heterogeneity is to exclude growers of medium ability. The second is the fact that heterogenizing some tournament groups will require homogenizing other groups.

Using the structural estimates and the recovered abilities for each grower, we can examine how heterogeneous the observed tournaments are. We find that the standard deviation of the abilities for all 356 growers is 0.0538 and the within the groups standard deviation ranges from 0.0303 to 0.0750, with an average of 0.0474. On the other hand, the between groups standard deviation (deviation of the means of abilities from different tournaments) is only 0.0099. These results indicate that given the pool of growers, the observed tournaments are fairly heterogeneous already. This implies that poultry companies might already be aware of the benefits of group heterogeneity and have exploited it to the greatest extent possible.

SUPPLEMENTARY MATERIALS

Proof and Analysis: This supplement consist of the proof of Proposition 1, the solutions to the principal's problem, the identification proof, and the detailed analysis of the alternative specification. (online_supplement.pdf)

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