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Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions

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Motivated by several interesting features of the highway mowing auction data from the Texas Department of Transportation (TDoT), we study three competing procurement auction models with endogenous entry. Our entry and bidding models provide several interesting implications. For the first time, we show that even within an independent private value paradigm, as the number of potential bidders increases, bidders' equilibrium bidding behaviour can become less aggressive, and the expected procurement cost may rise because the "entry effect" is always positive and may dominate the negative "competition effect". We then develop structural models of entry and bidding corresponding to the three models under consideration, controlling for unobserved auction heterogeneity, and use the recently developed semi-parametric Bayesian estimation method to analyse the data. We select the model that best fits the data, and use the corresponding structural estimates to quantify the "entry effect" and the "competition effect" with regard to the individual bids and the procurement cost.

1. INTRODUCTION

In this paper, which is motivated by several interesting features of the highway mowing auction data we collected from the Texas Department of Transportation (TDoT), we study three competing procurement auction models with endogenous entry. We then develop a fully structural econometric framework that is derived from our game-theoretic models of entry and bidding, and that also allows for controlling for unobserved auction heterogeneity. We develop a semi-parametric Bayesian method to estimate the underlying structural elements, namely, the entry cost distribution, the bidders' private cost distribution, and the distribution of the unobserved heterogeneity, and apply it to conduct a detailed structural analysis of the procurement auction data from TDoT.

Our theoretical models of entry and bidding are motivated by the strong evidence of entry behaviour in the dataset: on average only about 28·05% of the potential bidders (those who request official bidding proposals) actually submit their bids. Endogenous entry due to participation costs such as the cost involved in acquiring information and preparing for bidding as well as the opportunity cost in bidding (and winning) in one auction has been documented in recent empirical work (Bajari and Hortaçsu (2003) for eBay auctions, Athey, Levin and Seira (2004) for timber auctions). Theoretical models of entry, on the other hand, have been developed since the 1980s with Samuelson (1985) and McAfee and McMillan (1987*a*) studying pure strategies of entry, and Levin and Smith (1994) and McAfee, Quan and Vincent (2002) studying mixed strategies. As in Levin and Smith (1994), our first model assumes that each potential bidder has the same entry cost and adopts a symmetric mixed strategy in deciding whether to incur a cost in order to acquire his signal and submit a bid. Our model, on the other hand, differs from Levin and Smith (1994) in that while Levin and Smith (1994) assume that actual bidders know the number of actual bidders at the time of bidding, we relax this assumption.[1] Our second model is along the same lines as the first one but purifies the entry strategy by assuming that potential bidders draw different entry costs from the same distribution. In contrast to the first two models, which can be viewed as a two-stage game because potential bidders first decide whether to enter before they draw their private costs, our third model is a one-stage model similar in spirit to Samuelson (1985) in that potential bidders are assumed to have learned their private costs before entry. We derive some interesting model implications that are robust to the three models. For the first time, we show that even within the independent private value (IPV) paradigm, as the number of potential bidders increases, bidders' equilibrium bidding behaviour can become less aggressive and the expected procurement cost can rise.[2] Thus, increasing competition may not always be desirable for the government, meaning that our result, while somewhat counter-intuitive and surprising because of the pure IPV paradigm under consideration, can have important policy implications. This result can also be used to test empirically whether entry is an important part of the decision-making process. Notably, the relationship between bids and the number of potential bidders has been an important issue pursued in the theoretical and empirical literature, but has been limited to the framework in which there is no entry. In this framework, it has been shown that while in the IPV paradigm the relationship is monotone, this may not be the case in a common value (CV) model because of the interaction between the "competition effect" and the "winner's curse effect" (Bulow and Klemperer, 2002), and in an affiliated private value model due to the opposite "competition effect" and the "affiliation effect" (Pinkse and Tan, 2005). Our result is driven by the interaction of two opposite effects: the "competition effect" and the "entry effect". While the "competition effect" is always negative as usual, the "entry effect" is always positive. This positive "entry effect" suggests to a winning bidder that he may have overestimated the intensity of entry.

To analyse the data we have collected and also to provide a general framework within which entry and bidding are jointly modelled, we develop a fully structural framework jointly modelling entry and bidding based on the theoretical models we study. Our structural approach controls for unobserved auction heterogeneity, which is necessary since in procurement auctions, bidders

1. In contrast to the previous work that assumes an (exogenously) varying number of actual bidders and studies revenue and welfare implications (McAfee and McMillan, 1987*b*; Matthew, 1987; Harstad, Kagel and Levin, 1990; Levin and Ozdenoren, 2004), the uncertain number of actual bidders in our model is endogenous as a result of the endogenous entry.

2. In studying high-bid auctions with entry assuming that actual bidders know the number of actual bidders at the time of bidding, Levin and Smith (1994) show that under the optimal mechanism for the auctioneer where the auctioneer's expected revenue is maximized, and hence, the optimal entry is induced, the expected revenue (that is the expected winning bid) decreases with an increase in the number of potential bidders when the number of potential bidders grows beyond some cut-off point. Our result, on the other hand, does not require the optimal mechanism for the auctioneer. Thus, our result on the relationship between the procurement cost and the number of potential bidders complements Levin and Smith's result, whereas our result on the relationship between the individual bids and the number of potential bidders is the first established in the literature, to the best of our knowledge.

have access to information about the auctioned projects, some of which may not show up in the data.[3] From an econometric viewpoint, failing to control for unobserved auction heterogeneity in empirical research could severely bias the estimates for the structural parameters, and hence, yield misleading policy conclusions and recommendations, as shown in Krasnokutskaya (2002).

It is worth noting that joint modelling of entry and bidding is on one hand a modelling issue and on the other hand challenging from the perspective of econometric implementation, while controlling for unobserved auction heterogeneity is mainly an important econometric issue. This paper aims to provide a unified framework to address these two issues in structural inference of auctions. We achieve this objective by parameterizing the bidders' private cost distribution and the entry cost distribution, leaving the distribution of the unobserved heterogeneity term unspecified, and adopting the recently developed semi-parametric Bayesian method to estimate the underlying structural elements. The semi-parametric Bayesian method using a Dirichlet process prior is introduced by Lo (1984) and Ferguson (1983), with its recent development stimulated by the advances in the Markov chain Monte Carlo (MCMC) method (e.g. Escobar and West, 1995). While this method has been recently adopted by econometricians in addressing various issues (Hirano (2002) for estimation of linear autoregressive panel data models, Chib and Hamilton (2002) for analysis of longitudinal data treatment effect models, among others), our paper demonstrates its considerable advantage in making complicated structural models tractable.[4]

Jointly estimating fully structural models of entry and bidding has been a challenging problem because of the complexity involved with the joint models of entry and bidding, as well as the presence of latent variables. We are able to circumvent the difficulties by using the data augmentation techniques that enable us to develop a full Monte Carlo Markov chain to estimate the parameters in the structural model and the distribution of the unobserved heterogeneity. This contributes to the literature of structural analysis of auction data as, to the best of our knowledge, our paper is the first one that adopts a fully structural framework for entry and bidding, while controlling for unobserved auction heterogeneity.[5]

We use our structural approach to analyse the TDoT data by estimating the three models under consideration. We then use both in-sample and out-of-sample MSEPs to select the model that best fits the data. It turns out that our mixed strategy model is selected. Our results indicate that there is a significant effect from the unobserved heterogeneity. We find that on average the entry cost is about 13·8% of the private cost, or 8% of the winning bid, and hence, is a significant part in the bidders' decision-making process and a theoretical model or empirical analysis that ignores the entry effect may lead to poor policy recommendations.[6]

3. For example, in our dataset we do not observe the information on road conditions that may affect bidders' decision.

4. Bajari (1997) first used parametric Bayesian methods to estimate asymmetric auction models. Subsequent work by Bajari and his co-authors also uses parametric Bayesian methods to estimate auction models.

5. Bajari and Hortaçsu (2003) as well as Athey, Levin and Seira (2004) are important developments in taking into account bidders' participation when using the structural approach to analyse auction data, both using a reduced form specification for modelling entry while estimating a structural bidding model. Li (2005) derives moment conditions implied by the entry and bidding model in Levin and Smith (1994) when reserve prices are binding, which can be used for estimation and testing. He also suggests using a flexible reduced form specification for the entry probability. On the other hand, Haile, Hong and Shum (2002) develop tests for common values in first-price auctions allowing for entry and unobserved auction heterogeneity.

6. As in Athey, Levin and Seira (2004) and Bajari and Hortaçsu (2003), we are able to recover the entry cost in dollar terms using the equilibrium conditions and structural estimates. As a result, the structural approach in entry and bidding of auctions with incomplete information allows one to recover firms' profit functions in a similar spirit to the pioneering work by Bresnahan and Reiss (1990) and Berry (1992) who study entry in monopoly/oligopoly markets using game theoretic models with complete information.

We use the estimates of the structural parameters in the mixed strategy model to conduct counterfactual analysis by quantifying the effects of the number of potential bidders on the individual bid as well as on the procurement cost. We find that the "entry effect" is positive and the "competition effect" is negative. Moreover, the positive "entry effect" actually dominates the negative "competition effect" in our data and, hence, policies that encourage more potential bidders might cause the government's procurement costs to increase. We also quantify the savings for the government with regard to the procurement cost when the entry cost is reduced.

The empirical analysis of the procurement auctions conducted in this paper contributes to the growing literature on empirical analyses of procurement auctions using the structural approach. For example, see Bajari and Ye (2003) for detecting collusion, Hong and Shum (2002) for assessing the winner's curse, Jofre-Bonet and Pesendorfer (2003) for measuring the effect of capacity constraints in dynamic procurement auctions, Krasnokutskaya (2002) for measuring the effect of unobserved auction heterogeneity in asymmetric first-price IPV auctions, and Krasnokutskaya and Seim (2006) for measuring the effect of bid preference programmes and participation in highway procurement auctions.[7] Most of the work, while studying different aspects of procurement auctions, however, has assumed exogenous participation; a notable exception is Krasnokutskaya and Seim (2006). Our empirical analysis, on the other hand, is based on a fully structural framework for entry and bidding, and quantifies the effect of entry in the bidders' decision process, as well as the role of the unobserved auction heterogeneity using the selected model among the three competing models. In particular, the structural approach adopted here enables us to assess the "entry effect" and the "competition effect", as well as to quantify the relationship between the procurement cost and the entry cost, and hence, can be useful for making policy recommendations.

This paper is organized as follows. Section 2 presents the dataset that will be analysed to motivate the problem and to discuss the relevance of the theoretical models developed later. Section 3 is devoted to proposing the three models for entry and bidding, and deriving some comparative statics that are robust to the three models. Section 4 carries out several reduced-form empirical tests of the theoretical model using our data. Section 5 develops the structural framework for joint modelling of entry and bidding from the theoretical models proposed in Section 3 and proposes the semi-parametric Bayesian MCMC estimation algorithm to estimate the structural models. Estimation and model selection results are presented in Section 6. Section 7 carries out counterfactual analyses to quantify the "entry effect" and the "competition effect" on the individual bid and the procurement cost as well as the savings for the government when the entry cost is reduced. Section 8 concludes. The technical proofs and algorithms are included in the Appendix, which is separated from the main text of the paper and available at the journal website as supplementary material to the paper.

2. DATA

The data used in this study is from the highway construction and maintenance procurement auctions held by the TDoT between January 2001 and December 2003. We focus on a particular type of highway maintenance job, which is called "mowing highway right-of-way". We select this type of auctions for two reasons. First, this type of job is the single most frequently held auction in the sample period we study. Second, this type of auction is relatively simple

7. Another strand is to use reduced-form tests to test for collusion in procurement auctions. Porter and Zona (1993) represents this line of research.

compared with other big construction projects. These highway mowing projects usually involve a main task, which is called "type-II full width mowing", with several additional tasks such as strip mowing, spot mowing, litter pickup and disposal, sign installation and so on.

TDoT advertises projects three to six weeks prior to the letting date. Advertisement usually includes a short description of the project including the location, the completion time, the engineer's estimate and the estimated quantity for each task involved. In order to become an eligible bidder for a certain project, the contractor must first become a qualified bidder, which is based on the contractor's financial statement and is reviewed on a yearly basis.[8] TDoT maintains a master list for these qualified bidders, which lists all the qualified bidders in any sub-industry of the highway construction and maintenance industry in Texas. Second, those qualified contractors interested in the project must obtain a detailed project plan and the official bidding proposal from TDoT no later than 21 days prior to the letting date. TDoT maintains a bidders' list of those contractors who have requested the official bidding proposal for the project. We observe from the data that for a certain project, only (and usually all of) those contractors in the mowing sub-industry who are located in the same or nearby counties as where the job is would request the official bidding proposals. Therefore, we treat the bidders on the bidders' list for a certain project rather than the bidders on the master list as the potential bidders. Bids have to be submitted before the bid opening time in a sealed envelope.

The information flow of the auction mechanism is as follows. First, after a project advertisement is posted, bidders learn the auction specifics. During 21 days prior to the bid submission deadline, bidders learn how many potential bidders have already requested the official bidding proposal. Then they prepare and submit bids by the bid opening time without the knowledge of the number of actual bidders. Finally, all the bids and actual bidders' identities are revealed after the bid opening.

The dataset consists of 553 projects, with a total of 1606 bids. The information we observe from the data includes the letting date, the location (district, county, and highway), the tasks involved, the quantity of each task, the identity of all the planholders (potential bidders on the list of bidders), the identity of all the bidders who actually submit their bids (actual bidders), the actual bidding amount, the completion time, the amount of guaranty money, and whether it is a state or local maintenance job.

Table 1 gives the summary statistics of the data and several other quantities of interest. First, it is worth noting that entry behaviour is present in our dataset. For the auctions in our dataset, on average, an auction has about 11 potential bidders, but the number of actual bidders is only about 2·90. That only 28·05% of the potential bidders actually submit their bids later is strong evidence of bidders' entry behaviour. While requesting for a bidding proposal is free, acquiring signals and preparing for a bid is not. The entry behaviour indicates that bidders rationally take the entry cost into account and decide whether to enter into the final bidding process.

The mean of the engineer's estimate for these auctions is $165,348·90, which is very close to the mean of the bids, that is, $165,382·20. This shows that the engineer's estimate is the key determinant of bidders' bids.[9] On average, these projects last for about four months. Naturally, longer duration of the project will increase bidders' bids for the project. The size of the main task, that is, the "type-II full width mowing", is 7302·94 acres on average, with an average

8. For certain types of relatively simple projects like the mowing job studied in this paper, a contractor does not need to submit a financial statement to become qualified. Instead, they submit a bidder questionnaire to become qualified.

9. These numbers also imply that the financial stakes are quite high for bidders given that the potential bidders are usually individual contractors or two- or three-person small firms. Therefore, entry decision can be important.

TABLE 1

Summary statistics

Variable	Explanation	Obs	Mean	Standard deviation
Estimate	Engineer's estimate	553	165348·90	130782·30
Bids	Bids	1606	165382·20	133721·70
Day	Number of working days	553	122·72	120·07
Full	Acreage of full width mowing	553	7302·94	5219·91
Other	Acreage of other mowing	553	1987·64	2803·50
Items	Number of items	553	2·01	0·82
State	1 if it is a state job	553	0·11	0·31
Interstate	1 if it is on an interstate highway	553	0·23	0·42
Backlog	Bidder backlog	1606	−0·04	1·75
Potential	Number of potential bidders	553	11·08	3·47
Actual	Number of actual bidders	553	2·90	1·17
Entry	Actual/potential	553	0·28	0·12
Highbids	1 if the bid is higher than estimate	1606	0·44	0·50
Highwinner	1 if the winning bid is higher than estimate	553	0·22	0·41

of 1987·64 acres of other mowing jobs. If bidders have different costs for completing the two different kinds of jobs, then different composition of the jobs will lead bidders to submit different bids even though the total acreages of the mowing jobs are similar. The mean number of tasks in one mowing job is 2·01. Around 11% of the jobs are auctioned by the state agency and 23% of the jobs are on an interstate highway. We distinguish the state job from the local job because we expect that the two agencies may have different requirements for bid preparation, and hence, the entry cost may differ and hence the bidders change their bids accordingly. Also, a job on an interstate highway may cause bidders to bid more since it is both hard to transport equipment to an interstate highway and to complete the job on an interstate highway due to the high volume of traffic.[10]

Although TDoT states in the bidding rules that it will reject the contract if "the lowest bid is higher than the Department's estimate and re-advertising the project for bids may result in lower bids", in reality (at least in the auctions in our data), we do not see that this rule is enforced. In our dataset, we observe 704 bids higher than the Department's estimate, which account for about 43·86% of the entire sample. For 120 projects the winning bid is higher than the Department's estimate. These facts suggest that this rule has been ignored by bidders when making their decisions as it has never been implemented. The assumption of no binding public reserve price is justified in this environment. In effect, the auctions in our study can be considered as first (low)-price sealed-bid auctions without binding public reserve prices with the project awarded to the bidder with the lowest bid.

A possible concern is firms' capacity constraints that could cause a backlog, and thus, asymmetry across, and dynamic behaviour of bidders in, bidding as studied in Jofre-Bonet and Pesendorfer (2003). To address this issue, following Jofre-Bonet and Pesendorfer (2003), we define the backlog as the amount of work measured in dollars that is left remaining to be done from previously won projects. The backlog variable is constructed in the following way: for

10. These statistics of the projects, such as the average duration of four months, large sizes of the main tasks along with multiple side jobs, mean that the projects are significant and complex for the bidders who are usually individuals or two- or three-person firms and thus justify significant entry cost related to acquiring signal/information and bid preparation.

every contract previously won, we calculate the amount of work measured in dollars that is left to be done by taking the initial size of the contract and multiplying it by the fraction of time that is left until the project's planned completion date. For example, if the auction date is 23 May 2001 and the number of working days is 120, then the planned completion date is 20 September 2001.[11] On the basis of this calculation, for each bidder, we determine the total amount of work measured in dollars that is left to be done on any day during the sample period. We standardize the backlog variable by subtracting the bidder specific mean (calculated using daily observations) and dividing this difference by the bidder-specific standard deviation.[12] The resulting backlog variable is a number that is comparable across bidders. There is substantial variation in the backlog variable. On average, for about 11·5% of the observations, a bidder has a backlog within 0·1 standard deviation from the mean of the backlog variable, whereas for about 4·48% of the observations, the bidder is about two standard deviations above or below the mean of the backlog variable.

Lastly, there are about 13 auctions in our data having only one actual bidder. This accounts for about 2·35% of the entire sample. Our entry and bidding models will allow for the presence of only one actual bidder, as can be seen later.

3. GAME-THEORETIC MODELS FOR ENTRY AND BIDDING

In this section, we study three competing game-theoretic models of entry and bidding in procurement auctions and derive their equilibrium properties. The government lets a single and indivisible contract to N firm contractors who request the official bidding proposals and become potential bidders. While our model and econometric method can be readily adopted to the case where government announces a reserve price before the auction and the reserve price is effectively binding, we will focus on auctions without reserve prices to be consistent with the data under study. Each potential bidder is risk-neutral with a disutility equal to his private cost c of completing the contract, which is drawn from a distribution $F(\cdot)$ with support $[\underline{c}, \overline{c}]$. $F(\cdot)$ is twice continuously differentiable and has a density $f(\cdot)$ that is strictly positive on the support. In the IPV paradigm, when forming his bid, each actual bidder knows his private cost c, but does not know others' private costs. On the other hand, each bidder knows that all private costs are independently drawn from $F(\cdot)$, which is common knowledge to all bidders. As a result, all bidders are identical *a priori* and the game is symmetric.[13]

Finally, since we assume that there is no binding reserve price, as consistent with the TDoT data, and at the same time, we do observe some auctions with only one actual bidder, it is important to take these two aspects into account to rationalize bidders' behaviours. In first (high)-price sealed-bid auctions without (binding) public reserve prices, there is a unique finite Bayesian–Nash equilibrium bidding strategy with an uncertain number of actual bidders. This is not the case, however, for the first (low)-price sealed-bid auctions as considered here. This

11. Jofre-Bonet and Pesendorfer (2003) use the actual completion date to construct the backlog variable for most of their observations in their study. They only use the planned completion date for those projects that did not finish by the end of the sample period. This is not possible for our study here as the actual completion date is not observed.

12. For 27 bidders, the backlog variable takes the value of 0 for all days and, hence, both the bidder-specific mean and standard deviation are 0. For these bidders, the standardized backlog variable is set to be 0 for all days.

13. The assumption that bidders are symmetric can be justified as follows. First, in our data, the winning rate is roughly the same across all bidders. Second, bidders for the mowing jobs are usually individual contractors or two- or three-person small firms and, hence, the differences between bidders are small in terms of efficiency, capacity and other production factors. Moreover, as shown in the probit analysis of potential bidders' entry decisions, and the regressions of bids and winning bids in Section 4, the backlog variable is insignificant, meaning that bidders' different capacity constraints do not affect their entry decisions and bids.

is because for any rational bidder, if he knows that there is no binding public reserve price, and there is a non-zero probability for him to be the single actual bidder, no matter how small this probability is, his optimal strategy is always to bid infinity. Such a striking difference between high-bid and low-bid auctions, while important, has not been noticed in the literature. Therefore, to rationalize bidders' behaviours so that we can preserve a finite strictly increasing Bayesian–Nash equilibrium bidding strategy in our case, we assume that each potential bidder has a common belief that if he turns out to be the only actual bidder in the auction, he has to compete with the government who will draw its private cost from the same distribution $F(\cdot)$. Otherwise, if he turns out to be one of at least two actual bidders, he just competes with the other actual bidder(s). As a result, it is a common belief for each potential bidder that at each auction, if he decides to enter, he will face at least one other actual bidder, either the government (if he is the only actual real bidder) or another real bidder (if there are at least two actual real bidders).[14]

3.1. *Model 1: the mixed strategy entry model assuming that potential bidders do not draw their private costs until after entry*

Each potential bidder must incur an entry cost k to learn his private cost for completing the job and, hence, how to bid for the project. Thus, the auction is modelled as a two-stage game.

At the first stage, knowing the number of potential bidders N, each potential bidder learns the specifications of the project and the entry cost, calculates their expected profit from entering the auction conditioning on his winning, and then decides whether or not to participate in the auction and actually submit a bid. After the first entry stage, the n firm contractors who decide to enter the auction learn their own costs of completing the job, and submit their bids.

3.1.1. First-stage: mixed strategy entry. Denote $E\pi(b_{M_1}, c|q^*_{M_1})$ as the payoff for the actual bidder who optimally bids b_{M_1} using a Bayesian–Nash equilibrium strategy given his own cost c, the unique equilibrium entry probability $q^*_{M_1}$ and the belief that there must be at least two bidders (including himself) in the auction, as well as conditioning on that he is the winner. Extending Levin and Smith's (1994) symmetric mixed strategy entry model for high-bid auctions, we assume that there is a unique probability $q^*_{M_1}$ such that all the potential bidders have the same probability $q^*_{M_1}$ to enter the auction. The expression, $q^*_{M_1}$, is determined from the equilibrium where the *ex ante* expected payoff is equal to the entry cost k as given in the following equation

$$\int_{\underline{c}}^{\overline{c}} E\pi(b_{M_1}, c|q^*_{M_1}) f(c)dc = k. \tag{1}$$

3.1.2. Second stage: bidding. Following the entry stage using the mixed strategy described above, each actual bidder learns his private cost c for completing the job. As a

14. This assumption can be justified because the government does know the number of actual bidders, and also because we do observe auctions with only one actual bidder whose bid is finite. Without introducing a role played by the government, in the absence of a binding public reserve price, these finite bids from single actual bidders cannot be rationalized. In effect, our assumption is equivalent to introducing a random reserve price from the government when there is a single actual bidder. On the other hand, our assumption is not restrictive as it limits the government's role only to the auctions with a single actual bidder. Furthermore, this assumption is not essential for the theoretical results, as the same results can be shown to hold for the low-bid auction case with a binding public reserve price, or the high-bid auction case with/without a binding public reserve price.

result, his expected profit by bidding $b_{M_1,i}$ conditioning on his private cost c_i and winning the auction is

$$
\begin{aligned}
E\pi(b_{M_1,i}, c_i|q^*_{M_1}) &= \sum_{j=2}^{N} P_{B,q^*_{M_1}}(n=j|n\geq 2)(b_{M_1,i}-c_i)\Pr(b_{M_1,t}\geq b_{M_1,i}, \forall t\neq i|n=j) \\
&= (b_{M_1,i}-c_i)\sum_{j=2}^{N} P_{B,q^*_{M_1}}(n=j|n\geq 2)[1-F(s^{-1}_{M_1}(b_{M_1,i}|q^*_{M_1}))]^{j-1},
\end{aligned}
$$

where $s_{M_1}(\cdot|q^*_{M_1})$ is the strictly increasing equilibrium bidding strategy given the entry probability $q^*_{M_1}$, and

$$
P_{B,q^*_{M_1}}(n=j|n\geq 2) = \frac{\binom{N-1}{j-1}(q^*_{M_1})^{j-1}(1-q^*_{M_1})^{N-j}}{1-(1-q^*_{M_1})^{N-1}}
$$

is the probability that there will be j actual bidders in the auction from the actual bidder's point of view. Here we make the assumption that bidders have the belief that there will be at least two bidders (including himself) in the auction as discussed in Section 3.1, which is different from Levin and Smith (1994).

The Bayesian–Nash equilibrium of this model can be characterized by the following first-order condition:

$$
\begin{aligned}
&\frac{\partial\{s_{M_1}(c|q^*_{M_1})\sum_{j=2}^{N} P_{B,q^*_{M_1}}(n=j|n\geq 2)[1-F(c)]^{j-1}\}}{\partial c} \\
&= -c\sum_{j=2}^{N} P_{B,q^*_{M_1}}(n=j|n\geq 2)(j-1)[1-F(c)]^{j-2}f(c). \qquad (2)
\end{aligned}
$$

The unique solution to (2) subject to the boundary condition $s_{M_1}(\bar{c}) = \bar{c}$ is given as follows:

$$
s_{M_1}(c|q^*_{M_1}) = c + \frac{\sum_{j=2}^{N}[P_{B,q^*_{M_1}}(n=j|n\geq 2)\int_c^{\bar{c}}(1-F(x))^{j-1}dx]}{\sum_{j=2}^{N}[P_{B,q^*_{M_1}}(n=j|n\geq 2)(1-F(c))^{j-1}]}. \qquad (3)
$$

As a result, the equation determining the mixed strategy probability $q^*_{M_1}$ (1) can be rewritten as

$$
\int_{\underline{c}}^{\bar{c}}\sum_{j=2}^{N}[P_{B,q^*_{M_1}}(n=j|n\geq 2)\int_c^{\bar{c}}(1-F(x))^{j-1}dx]f(c)dc = k. \qquad (4)
$$

3.2. *Model 2: the pure strategy entry model assuming that potential bidders do not draw their private costs until after entry*

The underlying structure of the auction game remains the same as in Model 1 except that the entry cost is no longer assumed to be the same for all the potential bidders. Instead, the entry cost is now private to the potential bidders, meaning that each potential bidder knows his own entry cost but not others'. The private entry cost of a contractor is independently drawn from a distribution $H(\cdot)$ that is common knowledge to all bidders and with support $[\underline{k}, \bar{k}]$. $H(\cdot)$ is twice continuously differentiable and has a density $h(\cdot)$ that is strictly positive on the support.

3.2.1. First stage: pure strategy entry. Denote $E\pi(b_{M_2}, c|k^*)$ as the payoff for the actual bidder who optimally bids b_{M_2} using a Bayesian–Nash equilibrium strategy given his own cost c, the unique equilibrium entry cost cutoff point k^* and the belief that there must be at least two bidders (including himself) in the auction, as well as conditioning on that he is the winner. Different from Model 1, in this model, bidders no longer adopt a probability distribution for their entry decisions. Instead, they adopt a pure strategy. Specifically, in the equilibrium, there exists a unique entry cost cutoff point k^* such that for a potential bidder, namely, bidder i, he only enters the auction if his entry cost k_i is lower than k^* and otherwise remains outside of the auction. The value k^* is determined from the equilibrium where the *ex ante* expected payoff is equal to the entry cost cutoff point k^* as given in the following equation:

$$\int_{\underline{c}}^{\overline{c}} E\pi(b_{M_2}, c|k^*) f(c) dc = k^*, \tag{5}$$

implying the bidder with entry cost at k^* is indifferent between entering the auction and not entering.

3.2.2. Second stage: bidding. The resulting bidding strategy is very similar to that of Model 1. The only difference is that in Model 1, the equilibrium entry probability $q^*_{M_1}$ is defined as a potential bidder's mixed strategy for the entry decision, whereas in the current model, the entry probability denoted by $q^*_{M_2}$ is defined to be $\Pr(k_i < k^*) = H(k^*)$, the probability that a potential bidder (bidder i) draws a private entry cost that is lower than k^*. Formally, the unique Bayesian–Nash equilibrium bidding strategy subject to the boundary condition $s_{M_2}(\overline{c}) = \overline{c}$ is given as follows:

$$s_{M_2}(c|k^*) = c + \frac{\sum_{j=2}^{N} [P_{B,q^*_{M_2}}(n = j|n \geq 2) \int_c^{\overline{c}} (1 - F(x))^{j-1} dx]}{\sum_{j=2}^{N} [P_{B,q^*_{M_2}}(n = j|n \geq 2)(1 - F(c))^{j-1}]} \tag{6}$$

where $s_{M_2}(\cdot|k^*)$ is the strictly increasing equilibrium bidding strategy given the entry strategy,

$$P_{B,q^*_{M_2}}(n = j|n \geq 2) = \frac{\binom{N-1}{j-1}(q^*_{M_2})^{j-1}(1 - q^*_{M_2})^{N-j}}{1 - (1 - q^*_{M_2})^{N-1}}$$

is the probability that there will be j actual bidders in the auction from the actual bidder's point of view and $q^*_{M_2} = H(k^*)$.

As a result, the equation determining the equilibrium entry cost cutoff point k^* (5) can be rewritten as

$$\int_{\underline{c}}^{\overline{c}} \sum_{j=2}^{N} [P_{B,q^*_{M_2}}(n = j|n \geq 2) \int_c^{\overline{c}} (1 - F(x))^{j-1} dx] f(c) dc = k^*. \tag{7}$$

Also note that the right hand side of (7) is monotone increasing in k^*, whereas the left hand side of (7) is monotone decreasing in $q^*_{M_2}$ as given in (i) of Lemma 1 and thus monotone decreasing in k^* because $q^*_{M_2} = H(k^*)$. This implies that the equilibrium entry cost cutoff point exists and is unique.

It is worth noting that Model 2 is closely related to Model 1 as they both assume that potential bidders do not draw their private costs until after they decide to enter the auction, and at the time of bidding, the actual bidders do not know the number of actual bidders. Model 2, on the other hand, is obtained by purifying the mixed strategy entry in Model 1 through assuming that potential bidders' entry costs are different.[15] Thus, there is a subtle difference between the information structures and the equilibrium beliefs in Models 1 and 2. In particular, potential bidders are assumed to have a common entry cost (thus known to everyone) in Model 1, whereas in Model 2 they are assumed to have different entry costs, each of which is private information to the potential bidder himself, and they only know the distribution ($H(\cdot)$) from which the entry costs are drawn. As a result, the resulting entry probabilities $q^*_{M_1}$ and $q^*_{M_2}$ are different.

3.3. *Model 3: the pure strategy entry model assuming that potential bidders draw private costs before entry*

The underlying structure of the auction game remains the same as in Model 1 except that before deciding whether to enter the auction, potential bidders have already learned their private costs for completing the job. The model is similar to Samuelson (1985) with the difference that there is a public reserve price in Samuelson's model whereas there is no such a public reserve price in our model here to be consistent with the data. As a result, the boundary condition in our case is different from that in Samuelson (1985).

3.3.1. Cutoff point in private cost. A bidder with the cutoff point c^* is indifferent between submitting a bid or not. Suppose a bidder with this private cost enters. Then he will win only if no other bidder enters. Any contractor that enters will have a lower cost than him and therefore, will make a lower bid. Clearly, the optimal bid in this event is to bid infinity as there is no reserve price. Since we assume that each potential bidder has a common belief that there are at least two actual bidders if he enters, the bidder with c^* chooses his bid b^* to maximize his objective function

$$(b - c^*)\left[1 - F(s^{-1}(b))\right]$$

where $s(\cdot)$ denotes the strictly increasing Bayesian–Nash equilibrium bidding strategy with two bidders. The solution to this maximization problem is

$$b^* = c^* + \frac{\int_{c^*}^{\bar{c}} [1 - F(x)]\, dx}{1 - F(c^*)}. \tag{8}$$

Hence, the equilibrium cutoff point c^* is determined by the following zero-expected profit condition:

$$(b^* - c^*)\left[1 - F(c^*)\right]^{N-1} - k = \int_{c^*}^{\bar{c}} [1 - F(x)]\, dx \left[1 - F(c^*)\right]^{N-2} - k = 0 \tag{9}$$

because by definition, the bidder with cost c^* earns 0 profit. A potential bidder enters and submits a bid if his private cost is below c^*. We can also define an entry probability $q^*_{M_3}$ as $\Pr(c_i < c^*) = F(c^*)$ for bidder i with a private cost c_i.

15. Athey, Levin and Seira (2004) and Krasnokutskaya and Seim (2006) use models similar to Model 2 but assuming that the actual bidders know the number of actual bidders at the time of bidding in studying timber auctions and highway procurement auctions, respectively. Model 2 and models used in Athey, Levin and Seira (2004) and Krasnokutskaya and Seim (2006) can be viewed as following the approach in modelling entry with a pure strategy game pioneered by Bresnahan and Reiss (1990) with a difference that Bresnahan and Reiss (1990) study an entry game with complete information, whereas Athey, Levin and Seira (2004) and Krasnokutskaya and Seim (2006) study auctions with incomplete information.

3.3.2. Bidding strategy. For each actual bidder i, his expected profit by bidding $b_{M_3,i}$ conditioning on his private cost c_i and winning the auction is

$$\begin{aligned} E\pi(b_{M_3,i}, c_i) &= (b_{M_3,i} - c_i)\Pr(b_{M_3,t} \geq b_{M_3,i}, \forall t \neq i) \text{ if } c_i \leq c^* \\ &= (b_{M_3,i} - c_i)[1 - F(s_{M_3}^{-1}(b_{M_3,i}|c^*))]^{N-1} \text{ if } c_i \leq c^*, \end{aligned}$$

where $s_{M_3}(\cdot)$ is the strictly increasing equilibrium bidding strategy given the equilibrium cutoff point c^* from above.

The Bayesian–Nash equilibrium of this model can be characterized by the following first-order condition:

$$\frac{\partial\{s_{M_3}(c|c^*)[1 - F(c)]^{N-1}\}}{\partial c} = -c(N-1)[1 - F(c)]^{N-2} f(c). \tag{10}$$

The unique solution to (10) subject to the boundary condition $s_{M_3}(c^*) = b^*$ is given as follows:[16]

$$s_{M_3}(c|c^*) = c + \frac{\int_c^{c^*}[1 - F(x)]^{N-1}dx}{[1 - F(c)]^{N-1}} + \frac{[1 - F(c^*)]^{N-2}}{[1 - F(c)]^{N-1}}\int_{c^*}^{\bar{c}}[1 - F(x)]dx. \tag{11}$$

Note that a major difference between Model 3 and Models 1 and 2 is that potential bidders are assumed to have already known their private costs before the entry decision in Model 3, whereas they are assumed to only learn their private costs after the entry decision in Models 1 and 2. Therefore, Model 3 can be viewed as a one-stage model in which potential bidders submit bids if their private costs are below the cut-off point c^*, whereas Models 1 and 2 are for a two-stage game where potential bidders first decide on entry and then draw their private costs after entry and submit bids.[17]

3.4. *Comparative statics that are robust to the three models*

As discussed in the preceding subsection, the three models under consideration can all be used to model bidders' entry and bidding behaviours, though they differ in the assumptions on the underlying information structure, and entry strategies. It would be interesting to study the implications that hold for all these models, and use them to test whether the entry and bidding behaviours found in our data are consistent with the models. To this end, we will study several comparative statics that are robust to the three models and that are empirically testable.

The first relationship is the one between the entry probability $q^*_{M_\zeta}$ for $\zeta = 1, 2, 3$ and the number of potential bidders N. This result is not only interesting in its own right, but also constitutes a key intermediate step in studying how the number of potential bidders affects bidders' equilibrium bids in Models 1 and 2 as we will discuss later. Despite the complexity of our entry models, we are able to show that the relationship between $q^*_{M_\zeta}$ and N is negative while keeping everything else constant without imposing any conditions.[18]

16. The boundary condition $s_{M_3}(c^*) = b^*$ where b^* is given in (8) is clearly different from the boundary condition in Samuelson's (1985) model with a public reserve price, say r_0, which postulates that the bidding strategy at the cut-off point is equal to r_0.

17. Model 3 is similar in spirit to the (one-stage) dynamic model for a regular bidder studied in Jofre-Bonet and Pesendorfer (2003), as the latter allows the possibility of a regular bidder not submitting a bid either because the cost exceeds the upper bound of the reserve price, or because the cost arising from the future constraint outweighs the current gain from being awarded the contract.

18. Levin and Smith (1994) give a sufficient condition under which the relationship between the induced entry probability from the optimal auction mechanism and N is negative while everything else is kept constant. Our result gives such a relationship for any entry probability (not necessarily induced by the optimal auction mechanism) without imposing sufficient conditions.

Proposition 1. *In Model ζ, $\zeta = 1, 2, 3$, $\frac{\partial q^*_{M_\zeta}}{\partial N} < 0$. That is, as the number of potential bidders increases, holding everything else fixed, the bidders' equilibrium entry probability decreases.*

The intuition behind this relationship is straightforward. For instance, in Model 1, $q^*_{M_1}$ is determined by the entry equation (4). Since the entry cost k does not change, meaning that the expected gain for a bidder who enters into this auction also remains the same, and thus, the number of actual bidders the auction can support does not change, bidders lower their equilibrium entry probability when the number of potential bidders increases.

Now we turn to the relationship of our main interest, that is, the relationship between the equilibrium bid b_{M_ζ} and the number of potential bidders N. Because of the entry in the three models we consider, the total differentiation of b_{M_ζ} with respect to N can be decomposed into two parts: $db_{M_1}/dN = \partial b_{M_1}/\partial N + \partial b_{M_1}/\partial q^*_{M_1} * \partial q^*_{M_1}/\partial N$ in Model 1, $db_{M_2}/dN = \partial b_{M_2}/\partial N + \partial b_{M_2}/\partial k^* * \partial k^*/\partial N$ in Model 2, and $db_{M_3}/dN = \partial b_{M_3}/\partial N + \partial b_{M_3}/\partial c^* * \partial c^*/\partial N$ in Model 3. Such a decomposition indicates that the number of potential bidders, N, affects the equilibrium bids in two channels. The first channel is the usual "competition effect", as represented by $\partial b_{M_\zeta}/\partial N$. The second channel is the second term in the decomposition, which we label as the "entry effect". The next proposition gives results on the signs of these two effects as well as the relationship between bids and the number of potential bidders.

Proposition 2. *In Model ζ, $\zeta = 1, 2, 3$,*

(i) the competition effect $\frac{\partial b_{M_\zeta}}{\partial N} < 0$;

(ii) the entry effect, defined as $\frac{\partial b_{M_1}}{\partial q^*_{M_1}} * \frac{\partial q^*_{M_1}}{\partial N}$ in Model 1, $\frac{\partial b_{M_2}}{\partial k^*} * \frac{\partial k^*}{\partial N}$ in Model 2, and $\frac{\partial b_{M_3}}{\partial c^*} * \frac{\partial c^*}{\partial N}$ in Model 3, is always positive;

(iii) the relationship between b_{M_ζ} and N may not be monotone decreasing.

Part (i) in Proposition 2 establishes the negative "competition effect". This is an intuitive result since when everything else is fixed, as the number of potential bidders increases, bidders become more aggressive. The positive "entry effect" as established in part (ii) of Proposition 2 is also intuitive. For instance, in Model 1, as the number of potential bidders increases, holding everything else constant, the bidders' equilibrium entry probability decreases, as shown in Proposition 1. On the other hand, as the entry probability decreases, holding everything else constant, the equilibrium bidding strategy increases, which is given in Lemma 2 in Appendix B. This result can be seen intuitively since when the number of potential bidders is fixed but the bidders' equilibrium entry probability decreases, the expected number of actual bidders decreases. Therefore, actual bidders face less competition and they bid less aggressively. The resulting entry effect is thus positive. In the end, the relationship between b_{M_ζ} and N depends on the relative magnitudes of the positive "entry effect" and the negative "competition effect".

We now illustrate the non-monotone decreasing relationship between b_{M_ζ} and N by concrete examples. We choose the private cost distribution to be a $uniform(0, 1)$ and $k = 0{\cdot}1$ in Models 1 and 3, whereas in Model 2 the entry cost distribution is a $uniform(0, 0{\cdot}08)$. The equilibrium bidding strategies for Models 1, 2 and 3 are plotted for cases $N = 8$ and $N = 3$ in Figures 1, 2 and 3, respectively. Note that in all the three models, the equilibrium strategies are not monotone-decreasing with N. For example, as can be seen from Figure 1, when the private production cost c is greater than 0·09, the bidding line for $N = 8$ lies above the one for $N = 3$.

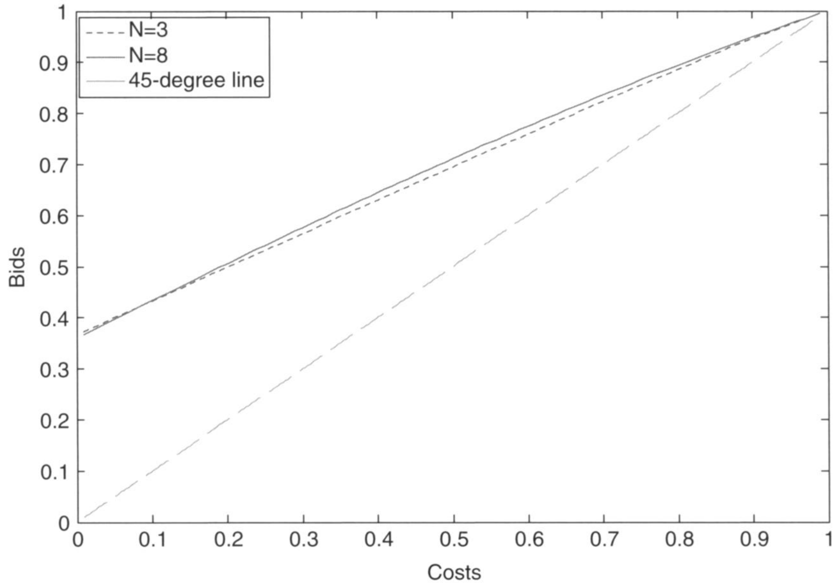

FIGURE 1

Example of non-monotone relationship between b and N for M_1

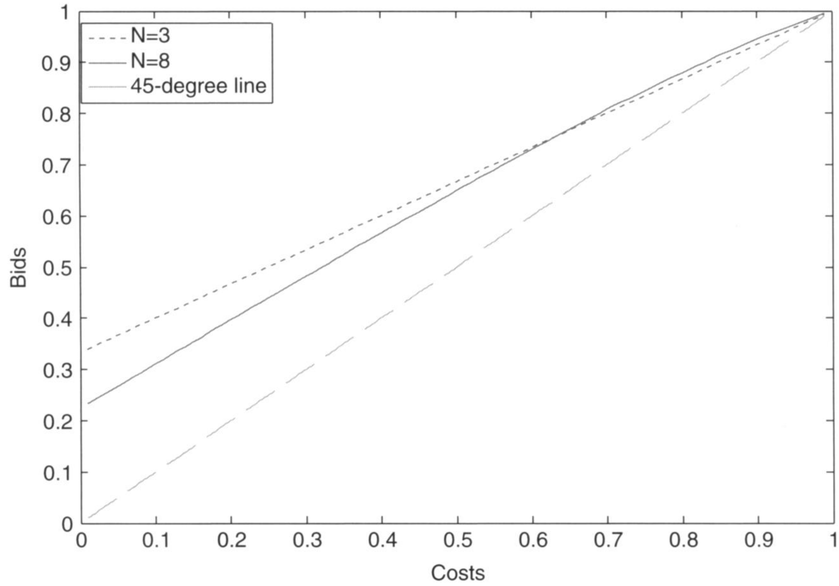

FIGURE 2

Example of non-monotone relationship between b and N for M_2

An interesting question is whether Proposition 2 holds also for the relationship between the expected winning bid (procurement cost) and the number of potential bidders. This is an important issue from an economic policy viewpoint as one of the central questions in the

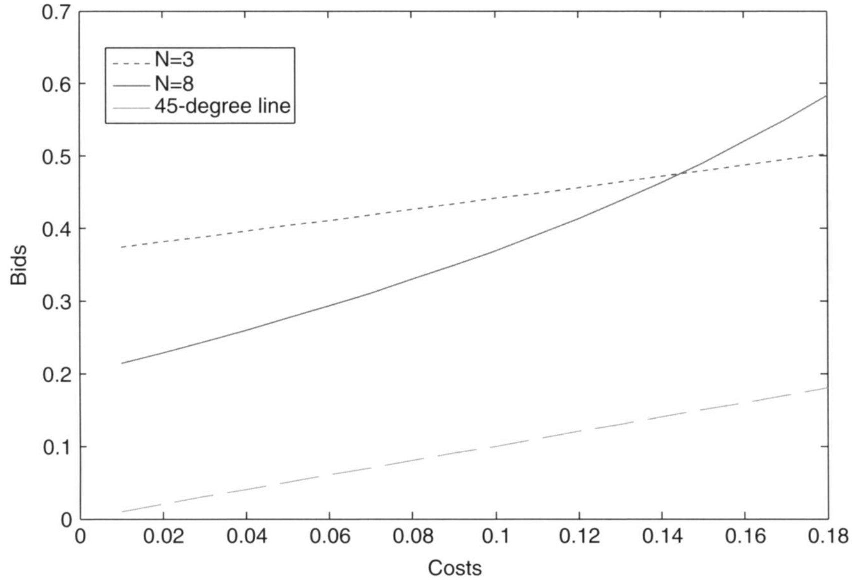

FIGURE 3

Example of non-monotone relationship between b and N for M_3

auction literature is to study how increasing the number of potential bidders can affect the expected procurement cost. The following proposition is concerned with this issue.

Proposition 3. *Denote the expected winning bid (procurement cost) in Model* ζ, $\zeta = 1, 2, 3$, *by* W_{M_ζ}. *Then*

(i) the competition effect $\frac{\partial W_{M_\zeta}}{\partial N} < 0$;

(ii) the entry effect, defined as $\frac{\partial W_{M_1}}{\partial q^*_{M_1}} * \frac{\partial q^*_{M_1}}{\partial N}$ *in Model 1,* $\frac{\partial W_{M_2}}{\partial k^*} * \frac{\partial k^*}{\partial N}$ *in Model 2, and* $\frac{\partial W_{M_3}}{\partial c^*} * \frac{\partial c^*}{\partial N}$ *in Model 3, is always positive;*

(iii) the relationship between W_{M_ζ} *and* N *may not be monotone decreasing.*

Again, the entry effect is positive and the competition effect is negative. Thus, it is possible that the relationship between the procurement cost and the number of potential bidders is not monotone decreasing. Proposition 3 has important policy implications. It establishes the relationship between the expected procurement cost and the number of potential bidders. Since at the equilibrium, the social welfare is equal to the social value of the project minus the expected procurement cost because the expected payoff for a bidder is zero, Proposition 3 implies that it is possible that social welfare can decline as the potential competition becomes more intense.

4. REDUCED-FORM EMPIRICAL TESTS OF THE MODEL

4.1. *The relationship between entry probability and N*

First, we use our procurement auction data to test whether there is a negative relationship between the equilibrium entry probability $q^*_{M_\zeta}$ and the number of potential bidders N, as predicted in Proposition 1.

While $q^*_{M_\zeta}$ is a conceptually defined random variable which is unobserved in our data, we can use the information on the number of actual bidders and the number of potential bidders to infer the relationship between $q^*_{M_\zeta}$ and N. Since n, the number of actual bidders, is a count variable, we assume that the number of actual bidders satisfies the following conditional mean condition:

$$E(n|\mathbf{x},N) = \exp(\mathbf{x}\boldsymbol{\zeta} + \rho \log N) = N^{\rho} \exp(\mathbf{x}\boldsymbol{\zeta}) \tag{12}$$

as in a Poisson regression model or generalized count regression models. Under this conditional mean assumption, using the Poisson quasi-MLE approach, one can consistently estimate ρ as well as $\boldsymbol{\zeta}$. Since (12) implies

$$E\left(\frac{n}{N}|\mathbf{x},N\right) = N^{\rho-1} \exp(\mathbf{x}\boldsymbol{\zeta}),$$

if the estimated ρ is less than 1, then as N increases, $\frac{n}{N}$, which is a good proxy for the entry probability, tends to decrease.

We include the logarithm of the engineer's estimate, the logarithm of the number of working days, the logarithm of the acreage of the type II full width mowing, logarithm of the acreage of other types of mowing jobs, the number of items in the job, whether the job is a state job and whether it is a highway job as the set of explanatory variables $\mathbf{x}$ in the Poisson regression model. Estimation results for the parameters $\boldsymbol{\zeta}$ and ρ are reported in Table 2. We note that the coefficient on the number of potential bidders is significant with a value of 0·4011, which is strictly less than 1. This result can be viewed as support of the inverse relationship between the entry probability and N. As the number of potential bidders increases, the equilibrium entry probability decreases.

We also go further to investigate this issue from another angle. We create a binary variable for each potential bidder that is 1 if he participates in the auction and 0 otherwise. Because of the possible unobserved auction heterogeneity we try to control for, we run a random-effects probit model for potential bidders' entry decisions. Table 3 reports the estimation results. The significant and negative coefficient on the number of potential bidders confirms the negative relationship between the entry probability and the number of potential bidders. Furthermore, the backlog variable is insignificant, meaning that potential bidders' different capacity constraints do not affect their entry decisions.

TABLE 2

Quasi-MLE Poisson regression estimates of number of actual bidders on explanatory variables

Variable	Estimate	Sd. error	t-statistic
Log(estimate)	−0·0200	0·0472	−0·42
Log(day)	0·0543	0·0386	1·41
Log(full)	0·0127	0·0078	1·64
log(other)	0·0230*	0·0063	3·63
Items	−0·1847*	0·0324	−5·69
State	−0·0581	0·0573	−1·01
Interstate	−0·0305	0·0388	−0·79
Log(potential)	0·4011*	0·0454	8·84
Constant	0·2698	0·4296	0·63
Log likelihood	−899·28		

*Significant at 5%.

TABLE 3

Random effects probit estimates of entry decision on explanatory variables

	Estimate	Sd. error	t-statistic
Log(estimate)	0·0343	0·0521	0·66
Log(day)	0·0023	0·0381	0·66
Log(full)	0·0118	0·0169	0·70
Log(other)	0·0197*	0·0085	2·32
Items	−0·1303*	0·0353	−3·69
State	−0·0409	0·0704	−0·58
Interstate	−0·0316	0·0436	−0·72
Backlog	−0·0103	0·0117	−0·88
Potential	−0·0395*	0·0054	−7·29
Constant	−0·5508	0·4606	−1·20
Log likelihood	−3356·98		

*Significant at 5%. Random effects model: $\Pr\left(\text{Entry}_{it} = 1\right) = \Phi\left(\mathbf{x}_{it}\boldsymbol{\lambda} + u_i\right)$.

4.2. *The relationship between bids and N*

Second, we empirically examine the relationship between the equilibrium bids and the number of potential bidders. We use log(bids) as our dependent variable. The set of exogenous variables include the backlog variable, in addition to those variables in $\mathbf{x}$.

Since we have 553 auctions with a total of 1606 bids and most of the auctions have more than one bid, this data structure is more like an unbalanced panel than a cross-section. We estimate a random effects panel data model to take advantage of this panel data like structure and to control for unobserved auction heterogeneity that is observed by bidders when making their decisions but unobserved by the econometrician.

The results are reported in Table 4. The number of potential bidders does not have a significant effect on bids at the 5% level with a t-statistic value of $-1{\cdot}14$. Furthermore, the error variance from unobserved heterogeneity accounts for 36% of the total error variance, supporting the existence of unobserved auction heterogeneity. The statistical insignificance of N in explaining bids can be viewed as consistent with the implication from our model as given

TABLE 4

Random effects estimates of log(bids) on explanatory variables

	Estimate	Sd. error	t-statistic
Log(estimate)	1·0267*	0·0172	59·53
Log(day)	−0·0263*	0·0121	−2·18
Log(full)	−0·0019	0·0051	−0·37
Log(other)	−0·0075*	0·0028	−3·37
Items	0·0271*	0·0118	2·31
State	0·0006	0·0240	0·02
Interstate	0·0200	0·0148	1·35
Backlog	−0·0004	0·0023	−0·19
Potential	−0·0020	0·0019	−1·10
Constant	−0·1590	0·1562	−1·02
Adjusted R^2	0·9327		

*Significant at 5%. Random effects model: $\log(\text{bids}_{it}) = \mathbf{x}_{it}\boldsymbol{\lambda} + u_i + \varepsilon_{it}$.

in Proposition 2. At the same time, it calls for a further structural analysis if one wants to quantify the "entry effect" and the "competition effect". Another interesting result from the regression is that the backlog variable has no significant effect on bids, meaning that in our data, capacity constraints do not play a significant role in determining bidders' bids.

4.3. *The relationship between winning bid and N*

Finally, we empirically examine the relationship between the winning bid and the number of potential bidders. We use log(winning bid) as our dependent variable and the same set of exogenous variables as used in the last subsection.

The result from the OLS regression is reported in Table 5. First, the backlog variable does not play a significant role in affecting winning bids. Second, again, the number of potential bidders does not have a statistically significant effect on the winning bid. Therefore, this result can also be regarded as consistent with the implication from our model as given in Proposition 3. Again, it calls for a further structural analysis if one wants to quantify the "entry effect" and the "competition effect" of the number of potential bidders on the winning bid.

5. (JOINT) STRUCTURAL INFERENCE OF THE ENTRY AND BIDDING EQUILIBRIUM MODELS

In this section, we derive structural models for entry and bidding from the game-theoretic models proposed in Section 3. We then propose to use the semi-parametric Bayesian method to jointly estimate the structural models with the data from TDoT. The semi-parametric Bayesian method enables us to jointly estimate the entry and bidding models while controlling for unobserved auction heterogeneity.

5.1. *Structural econometric framework*

In an econometric investigation, one often considers more than one auction, and the statistical inference is usually based on the assumption that the number of auctions approaches infinity. Therefore, possible heterogeneity across auctions needs to be taken into account. This issue

TABLE 5

OLS estimates of log(winning bid) on explanatory variables

Variable	Estimate	Sd. error	t-statistic
Log(estimate)	1·0399*	0·0184	56·58
Log(day)	−0·0299*	0·0130	−2·29
Log(full)	0·0023	0·0053	0·43
Log(other)	−0·0107*	0·0030	−3·60
Items	0·0212	0·0121	1·75
State	0·0073	0·0254	0·29
Interstate	0·0237	0·0156	1·52
Backlog	−0·0019	0·0036	−0·52
Potential	−0·0034	0·0020	−1·73
Constant	−0·4050*	0·1659	−2·44
Adjusted R^2	0·9544		

*Significant at 5%.

can be addressed by modelling the distribution of the private costs to depend on the heterogeneity of auctioned objects and, thus, to vary across auctions. Specifically, let $F_\ell(\cdot)$ denote the distribution of private costs for the ℓ-th auction, $\ell = 1, \dots, L$, where L is the number of auctions. Assume that $F_\ell = F(\cdot|\mathbf{x}_\ell, u_\ell, \boldsymbol{\beta})$, where $\mathbf{x}_\ell$ is a vector of variables that represents the observed auction heterogeneity, and u_ℓ is a variable representing the unobserved auction heterogeneity, both affecting bidders' costs, and β is an unknown parameter vector in $B \subset \boldsymbol{R}^{m_1}$. u is assumed to be independent of $\mathbf{x}$. Let $f(\cdot|\mathbf{x}_\ell, u_\ell, \beta)$ denote the corresponding density for bidders' private costs. Let N_ℓ denote the number of potential bidders and n_ℓ denote the number of actual bidders at the ℓ-th auction. An econometric issue arising from modelling entry is that the entry cost (for each potential bidder) at each auction is not observed. To resolve this, we assume that k_ℓ, $\ell = 1, \dots, L$, in Models 1 and 3, and $k_{i\ell}$, $i = 1, \dots, N_\ell$; $\ell = 1, \dots, L$, in Model 2, are randomly drawn from a distribution $H(\cdot|\mathbf{x}_\ell, u_\ell, \boldsymbol{\delta})$ with a density $h(\cdot|\mathbf{x}_\ell, u_\ell, \boldsymbol{\delta})$, where $\boldsymbol{\delta}$ is an unknown parameter vector in $\Psi \subset \boldsymbol{R}^{m_2}$.

5.2. *Specification and solving the equilibrium models*

For a structural auction model with endogenous entry and unobserved heterogeneity, a fully non-parametric estimation approach becomes infeasible and the classical likelihood or moment-based estimation approach becomes extremely complex.[19] In view of this, we propose to use the recently developed semi-parametric Bayesian estimation method to estimate the entry and bidding models jointly. This approach offers several considerable advantages. First, with data augmentation, the MCMC estimation method is relatively straightforward to implement. Second, the MCMC gives us the finite-sample properties of the resulting estimates. Third, incorporating a non-parametric unobserved heterogeneity component makes the specification of the model more flexible and hence the results more robust. Fourth, because of the parameter-dependent support arising from structural auction models, classical likelihood based inference becomes non-standard and computationally intensive as well.

We parameterize the density of contractors' private costs for completing the project as follows:

$$f(c|\mathbf{x}, u) = \frac{1}{\exp(\alpha + \mathbf{x}\boldsymbol{\beta} + u)} \exp\left[-\frac{1}{\exp(\alpha + \mathbf{x}\boldsymbol{\beta} + u)} c\right],$$

for $c \in (0, \infty)$. We further specify the entry cost density as follows:

$$h(k|\mathbf{x}, u) = \frac{1}{\exp(\gamma + \mathbf{x}\boldsymbol{\delta} + u)} \exp\left[-\frac{1}{\exp(\gamma + \mathbf{x}\boldsymbol{\delta} + u)} k\right], \tag{13}$$

for $k \in (0, \infty)$.

Note that while we employ the exponential distributions for specifying the conditional distributions of the private cost and entry cost given both observed and unobserved heterogeneities $\mathbf{x}$ and u, we leave the distribution of the unobserved heterogeneity u unspecified and will use

19. For non-parametric inference of auction models, see Guerre, Perrigne and Vuong (2000), Li, Perrigne and Vuong (2000, 2002), Athey and Haile (2002), Haile, Hong and Shum (2002), Hendricks, Pinkse and Porter (2003) among others that are thoroughly surveyed in Athey and Haile (2007). For classical likelihood or moment-based approaches, see Paarsch (1992), Donald and Paarsch (1993, 1996), Laffont, Ossard and Vuong (1995), Hong and Shum (2003), and Li (2005), to name a few.

the data to reveal the distribution of u.[20] Therefore, our specification and the resulting approach are semi-parametric in nature, and are expected to yield more robust results than a fully parametric specification. Of course, these specifications help in identifying the structural parameters and the distribution of the unobserved auction heterogeneity from a classical viewpoint.[21]

With these specifications, the equilibrium bidding strategies for the three models (3), (6) and (11) become

$$s_{M_\varsigma}(c) = c + \frac{\sum_{j=2}^{N} P_{B,q^*_{M_\varsigma}}(n=j|n\geq 2)\exp\left[-\frac{j-1}{\exp(\mu_1)}c\right]\frac{\exp(\mu_1)}{j-1}}{\sum_{j=2}^{N} P_{B,q^*_{M_\varsigma}}(n=j|n\geq 2)\exp\left[-\frac{j-1}{\exp(\mu_1)}c\right]}, \quad \varsigma = 1,2 \tag{14}$$

and

$$s_{M_3}(c) = c + \frac{\exp(\mu_1)}{N-1} + \frac{N-2}{N-1}\exp\left[\mu_1 - \frac{N-1}{\exp(\mu_1)}\left(c^* - c\right)\right],$$

where $\mu_1 = \alpha + \mathbf{x}\beta + u$. For the entry equilibrium, equations (4), (7) and (9) can be written as

$$\sum_{j=2}^{N}\left\{\frac{\binom{N-1}{j-1}(q^*_{M_1})^{j-1}(1-q^*_{M_1})^{N-j}}{1-(1-q^*_{M_1})^{N-1}}\frac{\exp(\mu_1)}{j(j-1)}\right\} = k,^{22} \tag{15}$$

$$\sum_{j=2}^{N}\left\{\frac{\binom{N-1}{j-1}(q^*_{M_2})^{j-1}(1-q^*_{M_2})^{N-j}}{1-(1-q^*_{M_2})^{N-1}}\frac{1}{j(j-1)}\right\} = -\log\left(1-q^*_{M_2}\right)\exp(\mu_2-\mu_1) \tag{16}$$

where $\mu_2 = \gamma + \mathbf{x}\boldsymbol{\delta} + u$ and

$$\exp\left[\mu_1 - \frac{N-1}{\exp(\mu_1)}c^*\right] = k,$$

respectively.[23]

20. The literature on parametric estimation of auction models has usually used extreme value distributions such as Weibull and exponential to specify private values distributions. See, *e.g.*, Paarsch (1992, 1997), Donald and Paarsch (1993, 1996), Haile (2001), Athey, Levin and Seira (2004). The specification of the exponential distributions for the conditional distributions of the private cost and the entry cost given both observed and unobserved heterogeneities is not as restrictive as it seems. This is because while this specification imposes that the conditional variance of the private cost (or the entry cost) be the square of the conditional mean given the observed and unobserved heterogeneities, this relationship no longer holds after the unobserved heterogeneity is integrated out. The same approach has been widely used in microeconometrics such as in modelling counts or durations while controlling for unobserved heterogeneity. Furthermore, in our setup, the distribution of the unobserved auction heterogeneity is unspecified, making our approach more robust. For modelling unobserved heterogeneity in micro data, see Heckman (2001) for an insightful discussion.

21. If either private costs or entry costs were observed, then the distribution of the unobserved heterogeneity in our setting could be non-parametrically identified following Horowitz (1999). In our case, however, we cannot observe private costs nor entry costs, but only observe bids and number of actual bidders, and the resulting structural models based on observables become highly non-linear. As a result, from a classical viewpoint, we have to assume the global identification of the structural elements as most of the work in the literature does, as local identification can be achieved through our model specification.

22. Note that the left hand side of this equation is a function of $q^*_{M_1}$ only given N and μ_1 and it goes to its maximum value $0{\cdot}5\exp(\mu_1)$ as $q^*_{M_1} \to 0$. Therefore, the entry cost equation implies that for this model, the entry costs in the observed auctions must satisfy the following restriction $k \leq 0{\cdot}5\exp(\mu_1)$, which has to be taken into account in the estimation.

23. Note that as $q^*_{M_2}$ increases from 0 to 1, the left hand side of (16) decreases monotonically from 0·5 to $\frac{1}{N(N-1)}$, whereas the right hand side of (16) increases monotonically from 0 to infinity. As a result, there exists a unique solution for $q^*_{M_2}$ between 0 and 1 in terms of model primitives.

5.3. *Semi-parametric Bayesian joint estimation of entry and bidding*

Our objective is to estimate the entry and bidding models jointly using both the observed numbers of actual bidders and bids. Joint estimation of the entry and bidding models is intractable, if not impossible, using the classical likelihood or moment-based methods because of the complexity of the two models and the presence of latent variables in both models. The Bayesian method, on the other hand, provides a computationally feasible alternative because of the use of the data augmentation and MCMC techniques. To make use of the semi-parametric Bayesian estimation and the MCMC algorithm, we first make some transformations. Let $e_{new} = \alpha + u$ represent the (un-normalized) unobserved heterogeneity term. Then $\mu_1 = \mathbf{x\beta} + e_{new}$. To fix ideas, we focus our discussion on Model 1 as the methods for estimating Models 2 and 3 are similar. With the private cost distribution specified above and equation (14) we can obtain the following implied distributions for the equilibrium bids in M_1:[24]

$$f(b|\mu_1) = \frac{1}{\exp(\mu_1)} \exp[-\frac{1}{\exp(\mu_1)}\varphi(b)]|\frac{\partial\varphi(b)}{\partial b}|, \tag{17}$$

for $b \in [\sum_{j=2}^{N} P_B, q_{M_1}^*(n=j|n\geq 2)\frac{\exp(\mu_1)}{j-1} / \sum_{j=2}^{N} P_B, q_{M_1}^*(n=j|n\geq 2), \infty)$, where $\varphi(b)$ is the inverse bidding function, and

$$\frac{\partial\varphi(b)}{\partial b} = \frac{\left[\sum_{j=2}^{N} P_B, q_{M_1}^* n=j|n\geq 2) \exp(-\frac{j-1}{\exp(\mu_1)}\varphi(b))\right]^2}{\left[\sum_{j=2}^{N} P_B, q_{M_1}^*(n=j|n\geq 2) \exp(-\frac{j-1}{\exp(\mu_1)}\varphi(b))\frac{1}{j-1}\right]\left[\sum_{j=2}^{N} P_B, q_{M_1}^*(n=j|n\geq 2) \exp(-\frac{j-1}{\exp(\mu_1)}\varphi(b))(j-1)\right]}. \tag{18}$$

Note that the lower support of b expressed here clearly indicates its dependence on the structural parameters, which violates the regularity conditions of the classical maximum likelihood estimation (Donald and Paarsch, 1993, 1996; Chernozhukov and Hong, 2004; Hirano and Porter, 2003). Also note that although equation (14) gives a complicated relationship between b and c, this relationship is monotone increasing. Moreover, it is computationally tractable to compute both $\varphi(b)$ and $\partial\varphi(b)/\partial b$, and hence to evaluate (17). Also, define

$$\mu_2 = \mu_1 - \mathbf{x\delta}^* - \gamma^*$$

where $\mathbf{\delta}^* = \beta - \mathbf{\delta}$, $\gamma^* = \alpha - \gamma$. we can rewrite (13) as

$$h(k|\mu_1, \mathbf{\delta}^*, \gamma^*) = \frac{1}{\exp(\mu_1 - \mathbf{x\delta}^* - \gamma^*)} \exp[-\frac{1}{\exp(\mu_1 - \mathbf{x\delta}^* - \gamma^*)}k], \tag{19}$$

for $k \in (0, \infty)$.

In M_1, using (15) and (19), because of the one-to-one (inverse) relationship between $q_{M_1}^*$ and k as discussed previously, the density for $q_{M_1}^*$ implied by the density of k is

$$\begin{aligned} & p(q_{M_1}^*|\mu_1, \mathbf{\delta}^*, \gamma^*) \\ = \; & h(k|\mu_1, \mathbf{\delta}^*, \gamma^*) \times |\frac{\partial k}{\partial q_{M_1}^*}| \times \boldsymbol{I}(k \leq 0{\cdot}5\exp(\mu_1)), \end{aligned}$$

24. Since the estimation procedures for the three models are similar to one another, only the estimation details for M_1 are presented in the main text. The estimation details for other models are collected in Appendix C.

where $\boldsymbol{I}(\cdot)$ is an indicator function and

$$\frac{\partial k}{\partial q^*_{M_1}} = \sum_{j=2}^{N} \left\{ \binom{N-1}{j-1} \frac{\exp(\mu_1)}{j(j-1)} \left[\begin{array}{c} \frac{(j-1)(q^*_{M_1})^{j-2}(1-q^*_{M_1})^{N-j}}{1-(1-q^*_{M_1})^{N-1}} + \frac{(N-j)(q^*_{M_1})^{j-1}(1-q^*_{M_1})^{N-j-1}}{1-(1-q^*_{M_1})^{N-1}} \\ -\frac{(N-1)(q^*_{M_1})^{j-1}(1-q^*_{M_1})^{2N-j-2}}{\left(1-(1-q^*_{M_1})^{N-1}\right)^2} \end{array} \right] \right\}.$$

Since the distributions of u and thus of $e_{new} = \mu_1 - \mathbf{x}\boldsymbol{\beta}$ are left unspecified, we use a non-parametric method to approximate e_{new}. Specifically, we use an infinite mixture of normals to approximate the unknown distributions. This is justified because Ferguson (1983) notes that any probability density function can be approximated arbitrarily closely in the L_1 norm by a countable mixture of normal densities.

$$g(\cdot) = \sum_{j=1}^{\infty} p_j \phi(\cdot|d_j, \sigma_j^2), \tag{20}$$

where $p_j \geq 0$, $\sum_{j=1}^{\infty} p_j = 1$ and $\phi(\cdot|d_j, \sigma_j^2)$ denotes the probability density function for a normal distribution with mean d_j and variance σ_j^2. Note that this is a different specification from the finite mixture of normals as used in Geweke and Keane (2001) since we do not have a prior number of components in the mixture of normals. Instead, we will use the Dirichlet process prior to carry out the Bayesian non-parametric density estimation and update the number of components in the infinite mixture of normals and the mean and variance for each component.

We use the data augmentation approach (Tanner and Wong, 1987) and include unobservables $\mu_{1,\ell}$ and $q^*_{M_1,\ell}$ in the algorithm drawing them at each iteration and for all observations. Our semi-parametric Bayesian estimation consists of two main parts. At each iteration, in the first part, using the augmented latent variables $\mu_{1,\ell}$ and $q^*_{M_1,\ell}$ and the current value of parameters, we can recover the unobserved heterogeneity terms through the relationship $e_{new,\ell} = \mu_{1,\ell} - \mathbf{x}_\ell\beta$. After recovering the unobserved heterogeneity terms, we can use a Bayesian approach explained in detail in the Appendix to estimate their densities with a Dirichlet process prior for the unknown densities. We update the number of components (denoted by m_c, say) in approximating mixture of normals and the mean and variance of the normal, denoted by d_ℓ and σ_ℓ^2 respectively for each ℓ. This approach was introduced by Lo (1984) and Ferguson (1983), with later work by Escobar and West (1995) among others discussing its computational issues. In the second part of each iteration, we update the model parameters and values for the latent variables. Denote $\boldsymbol{\Theta} = (\boldsymbol{\delta}^*, \gamma^*)$. We will use the following prior distributions: $\beta \sim Normal(\beta_0, B_0^{-1})$, $\boldsymbol{\Theta} \sim Normal(\theta_0, D_0^{-1})$, where β_0, B_0^{-1}, θ_0, D_0^{-1} are known parameters. $Normal(\beta_0, B_0^{-1})$ denotes a multivariate normal distribution with mean vector β_0 and covariance matrix B_0^{-1}. Denote $\Delta = (\beta, \boldsymbol{\Theta})$ and $\mathbf{x}^*_\ell = (1, \mathbf{x}_\ell)$. Then the joint posterior density of the parameters and unobservables $\mu_{1,\ell}$ and $q^*_{M_1,\ell}$ given the data and the distribution of the unobserved heterogeneity terms is

$$\begin{aligned} &\pi(\Delta, \{\mu_{1,\ell}, q^*_{M_1,\ell}\}_{\ell=1}^{L} | b, n, \{d_\ell, \sigma_\ell^2\}_{\ell=1}^{L}) \\ \propto\ & prior(\Delta) \times \prod_{\ell=1}^{L} p(b_{1\ell}, \ldots, b_{n_\ell \ell}|\mu_{1,\ell}, q^*_{M_1,\ell}) \times p(n_\ell|n_\ell \geq 2,\ q^*_{M_1,\ell}) \times p(q^*_{M_1,\ell}|\mu_{1,\ell}, \Delta) \\ & \times p(\mu_{1,\ell}|d_\ell, \sigma_\ell^2) \prod_{i=1}^{n_\ell} \boldsymbol{I}\left[b_{i\ell} \geq \sum_{j=2}^{N_\ell} P_{B}, q^*_{M_1,\ell}(n_\ell = j|n_\ell \geq 2) \frac{\exp(\mu_{1,\ell})}{(j-1)} \right] \end{aligned}$$

$$
\begin{aligned}
&\propto prior(\Delta)\\
&\times \prod_{\ell=1}^{L} \frac{1}{\exp(n_\ell \mu_{1,\ell})} \left[-\frac{1}{\exp(\mu_{1,\ell})} \sum_{i=1}^{n_\ell} \varphi(b_{i\ell}) \right] \prod_{i=1}^{n_\ell} \left| \frac{\partial \varphi(b_{i\ell})}{\partial b_{i\ell}} \right| \\
&\times \prod_{i=1}^{n_\ell} \boldsymbol{I} \left[b_{i\ell} \geq \sum_{j=2}^{N} P_B, q^*_{M_1,\ell}(n_\ell = j | n_\ell \geq 2) \frac{\exp(\mu_{1,\ell})}{(j-1)} \right] \\
&\times \frac{\binom{N_\ell}{n_\ell} (q^*_{M_1,\ell})^{n_\ell} (1 - q^*_{M_1,\ell})^{N_\ell - n_\ell}}{1 - (1 - q^*_{M_1,\ell})^{N_\ell} - N_\ell q^*_{M_1,\ell} (1 - q^*_{M_1,\ell})^{N_\ell - 1}} \\
&\times \frac{1}{\exp(\mu_\ell - \mathbf{x}^*_\ell \boldsymbol{\Theta})} \exp\left(-\frac{1}{\exp(\mu_{1,\ell} - \mathbf{x}^*_\ell \boldsymbol{\Theta})} k_\ell\right) \\
&\times \left| \frac{\partial k_\ell}{\partial q^*_{M_1,\ell}} \right| \times 1(k_\ell \leq 0{\cdot}5 \exp(\mu_{1,\ell})) \\
&\times \frac{1}{\sqrt{2\pi \sigma^2_\ell}} \exp[-0{\cdot}5 \sigma_\ell^{-2} (\mu_{1,\ell} - \mathbf{x}_\ell \boldsymbol{\beta} - d_\ell)^2]
\end{aligned}
$$

where[25]

$$
k_\ell = \sum_{j=2}^{N_\ell} \left\{ \frac{\binom{N_\ell - 1}{j-1} (q^*_{M_1,\ell})^{j-1} (1 - q^*_{M_1,\ell})^{N_\ell - j}}{1 - (1 - q^*_{M_1,\ell})^{N_\ell - 1}} \frac{\exp(\mu_{1,\ell})}{j(j-1)} \right\}.
$$

We construct our Markov chain blocking the parameters and the latent variables as $(\mu_{1,\ell}, q^*_{M_1,\ell})$, $\boldsymbol{\beta}$, $\boldsymbol{\Theta}$ and $(d_\ell, \sigma^2_\ell, m_c)$ with the full conditional distributions: $[\mu_{1,\ell}, q^*_{M_1,\ell} | \boldsymbol{\beta}, \boldsymbol{\delta}^*, \gamma^*, d_\ell, \sigma^2_\ell]$, $[\boldsymbol{\beta} | \mu_{1,\ell}, d_\ell, \sigma^2_\ell]$, $[\boldsymbol{\Theta} | \mu_{1,\ell}, q^*_{M_1,\ell}]$, $[d_\ell, \sigma^2_\ell, m_c | \mu_{1,\ell}, \boldsymbol{\beta}]$. The following steps summarize our algorithm with details given in Appendix C.II.

Sampling $(\mu_{1,\ell}, q^*_{M_1,\ell})$. Since the evaluation of the likelihood involves computing the inverse bidding function $\phi(b_{i\ell})$ at each iteration and for each bid, it is quite time consuming. In order to save computational time, we propose to block $(\mu_{1,\ell}, q^*_{M_1,\ell})$ together rather than to update them individually. Note also that the posterior density for $(\mu_{1,\ell}, q^*_{M_1,\ell})$ does not have a form that can facilitate a direct random draw from it. To overcome the associated computational problem, we propose to utilize the Metropolis–Hasting (MH) algorithm (Metropolis *et al*, 1953; Hastings, 1970) to draw from the density.

Sampling $\boldsymbol{\beta}$. Draw $\boldsymbol{\beta}$ given $\mu_{1,\ell}$, d_ℓ, σ^2_ℓ and its prior, which is a normal distribution with variance $\Lambda = \left(B_0 + \sum_{\ell=1}^{L} \sigma_\ell^{-2} \mathbf{x}'_\ell \mathbf{x}_\ell\right)^{-1}$ and mean $\bar{\boldsymbol{\beta}} = \Lambda \left(B_0 \boldsymbol{\beta}_0 + \sum_{\ell=1}^{L} \sigma_\ell^{-2} \mathbf{x}'_\ell (\mu_{1,\ell} - d_\ell)\right)$.

Sampling $\boldsymbol{\Theta} = (\boldsymbol{\delta}^*, \gamma^*)$. The full conditional density for $\boldsymbol{\Theta} = (\boldsymbol{\delta}^*, \gamma^*)$ is

$$
\begin{aligned}
\pi[\Theta | \mu_{1,\ell}, q^*_\ell] &= \exp\left[-(\boldsymbol{\Theta} - \theta_0)' D_0 (\boldsymbol{\Theta} - \theta_0)/2\right] \\
&\quad \times \prod_{\ell=1}^{L} \frac{1}{\exp(\mu_{1,\ell} - \mathbf{x}^*_\ell \boldsymbol{\Theta})} \exp\left(-\frac{1}{\exp(\mu_{1,\ell} - \mathbf{x}^*_\ell \boldsymbol{\Theta})} k_\ell\right).
\end{aligned}
$$

25. Note that here we modify the conditional distribution of the number of actual bidders to be a trucated binomial distribution from below at 2. This is because we concentrate on the subsample of auctions with at least two actual bidders in the structural estimation. This subset consists of 97·5% of the data; by the assumption of our model the government does not play any role in these auctions.

This posterior density does not allow one to make direct random draws. Again, we utilize the MH algorithm.

Updating d_ℓ, σ_ℓ^2 and m_c. Obtain $e_{new,\ell} = \mu_{1,\ell} - \mathbf{x}_\ell \boldsymbol{\beta}$ and update d_ℓ, σ_ℓ^2, and m_c using the non-parametric Bayesian estimation method as outlined in Appendix C.I.

These steps constitute one MCMC iteration. In the end, we will be able to obtain the posterior densities for $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\gamma - \alpha$, as well as the non-parametric density for e_{new}.[26]

6. RESULTS

In this section, we present the estimation results for the structural elements of the three joint bidding and entry models. We choose $\mathbf{x} = \{$log(estimate), log(day), log(full), log(other), items, state, interstate$\}$ to control for the observed auction heterogeneity. See the summary statistics and variable definitions in Table 1. For all the models, we centre priors for parameters $\boldsymbol{\beta} \sim N(\mathbf{0}_7, 10I_7)$ information on these parameters. The expression $\mathbf{0}_7$ denotes a vector of length 7 of 0's and I_7 is the 7-dimensional identity matrix. In the non-parametric component, we set $\mu_0 = 0$, $\tau_0 = 4$, $N_0 = 5$ unobserved heterogeneity. Finally, the tuning parameters are chosen to be $h_0 = 0{\cdot}5$, $h_1 = 0{\cdot}05$, $v_0 = v_1 = v_2 = 15$ and $\tau_2 = 1$ to obtain reasonable acceptance rates in the MH steps within the Gibbs Sampler.

We run the MCMC for 15,000 iterations, with an initial 5000 burn-in period. With the current setting of the tuning parameters, the average acceptance rates are 12·12% for sampling $(\mu_{1,\ell}, q^*_{M_1,\ell})$ and 34·03% for sampling $\boldsymbol{\Theta}$ in M_1, 15·25% for sampling $(\mu_{1,\ell}, q^*_{M_2,\ell})$ in M_2, 26·32% for sampling $(\mu_{1,\ell}, c^*_\ell)$ and 32·86% for sampling $\boldsymbol{\Theta}$ in M_3, and 21·77% for sampling $(\mu_{1,\ell}, q^*_{M_1,\ell})$ and 37·80% for sampling $\boldsymbol{\Theta}$ in M_1 without unobserved heterogeneity. The computation times for this algorithm are about 2·56 seconds for M_1, 2·36 seconds for M_2, 0·82 seconds for M_3 and 1·78 seconds for M_1 without unobserved heterogeneity, per draw on average on a Pentium®4 3·40 GHz processor. More than half of the computation time is from calculating the inverse bidding function. This computational burden is bearable considering the complexity of the estimation problem we try to solve.

6.1. *Parameter estimates and model selection*

We summarize the estimation results of the three models from the semi-parametric Bayesian algorithm in Table 6. Note that common to the three models, in the private cost distribution, the variable log(estimate) has a large positive effect on the mean of bidders' private cost for completing the job. The estimated coefficients on this variable from the three models have about the same magnitude around 0·95, indicating that the engineer's estimate is a good estimate for the costs of completing the job and bidders use this information to determine their own private costs for a contract. A1 unit increase in the log(estimate) will increase the mean of bidders' private cost by about 1·6 times.

Regarding the entry cost distribution, we also have interesting results. First, again common to the three models, the log(estimate) turns out to have a large positive effect on the entry cost. This is reasonable since contracts with higher values usually involve more tasks and, hence, more cost estimation work and more paper work for preparing the bids. Second, the estimates for the interstate dummy are also positive, meaning that mowing along an interstate highway in an auction can increase the entry cost mean.

While the aforementioned variables have similar effects in the three models, there are some variables such as log(day) having different estimates in terms of signs or/and magnitudes

26. Since $e_{new} = \alpha + u$ with $E[u] = 0$, we can also recover α from the estimated $E[e_{new}]$, and thus γ.

TABLE 6

Estimation results from semiparametric Bayesian algorithm

	Private cost distribution			Entry cost distribution		
Variable	M_1	M_2	M_3	M_1	M_2	M_3
Log(estimate)	0·9425	0·9812	1·0008	0·8330	1·0935	0·8287
	(0·0364)	(0·0357)	(0·0310)	(0·0514)	(0·1784)	(0·0522)
Log(day)	0·0004	−0·0071	−0·1171	0·0336	−0·1116	0·0023
	(0·0523)	(0·0420)	(0·0299)	(0·0771)	(0·1241)	(0·0778)
Log(full)	−0·0084	0·0111	0·0432	−0·0243	−0·0591	0·0054
	(0·0218)	(0·0150)	(0·0107)	(0·0306)	(0·0515)	(0·0249)
Log(other)	−0·0103	−0·0037	$-3{\cdot}2968 * 10^{-4}$	−0·0286	−0·0955	−0·0115
	(0·0121)	(0·0094)	(0·0068)	(0·0177)	(0·0309)	(0·0175)
Items	0·0583	$6{\cdot}6941 * 10^{-4}$	0·0162	0·1817	0·5011	0·0619
	(0·0526)	(0·0402)	(0·0280)	(0·0747)	(0·1342)	(0·0714)
State	0·0778	0·0231	−0·0896	0·2113	0·1334	0·1700
	(0·0999)	(0·0830)	(0·0552)	(0·1385)	(0·2507)	(0·1296)
Interstate	0·0409	0·0170	0·0114	0·0716	0·1009	0·0293
	(0·0672)	(0·0510)	(0·0368)	(0·0946)	(0·1540)	(0·0937)
Diff. in constants ($\alpha - \gamma$)	0·1833	0·9929	0·2877			
	(0·2357)	(1·5854)	(0·2249)			

Notes: Results are based on 15,000 draws following a 5000 initial burn-in period. For each variable, the first row contains the means and the second row contains the standard deviations of the MCMC sample draws.

across the three models. To assess the fit of our models and select the best model, we employ both the in-sample and out-of-sample mean squared error of predictions (MSEP) for the entry ratio variable $\frac{n}{N}$ and bid b, respectively, using the estimated structural parameters from the three models.[27] For the out-of-sample MSEP, we re-estimate the models using the first 400 auctions in our data, and then using the estimated structural parameters and the covariates of the other 140 auctions in the data, we predict $\frac{n}{N}$ and bid b for the 140 auctions. Results on MSEPs from both the in-sample and the out-of-sample procedures are reported in columns 2 to 4 in Table 7. The overwhelming evidence from these results is that Model 1 fits the data better than Models 2 and 3. Furthermore, we also calculate the same MSEPs for Model 1 without unobserved heterogeneity, with results reported in column 5 in Table 7. Comparison of the MSEPs between Model 1 with and without unobserved heterogeneity clearly favours the model with unobserved heterogeneity.

In view of these model fit results, we will focus our structural analysis of the data based on the estimates from Model 1 with unobserved heterogeneity. We use the structural estimates to find the entry cost in relation to the private cost by simulating the entry cost and the private cost from the corresponding estimated distributions for each auction. The average ratio between the entry cost and the private cost is about 13·8%, and the average ratio between the entry cost and the winning bid is about 8·3%. The finding that the entry cost is about 8% of the winning bid on average is comparable to the finding by Bajari, Hong and Ryan (2007) using the procurement data from the California Department of Transportation that the entry cost is about 5% of the winning bid on average. This finding also indicates that the entry cost is a

27. As pointed out by a referee, fundamentally the three models may not be distinguishable from each other. It thus becomes a statistical question in terms of model selection. Since the three models are non-nested, we resort to the MSEP, a model selection criterion that is commonly used in practice.

TABLE 7

Model fit (MSEP) results

	M_1	M_2	M_3	M_1 w/t Unobserved heterogeneity
In sample				
$\frac{n}{N}$	0.0039	0.0023	0.0093	0.0039
b	$0{\cdot}8098 * 10^9$	$0{\cdot}9624 * 10^9$	$1{\cdot}2401 * 10^{10}$	$0{\cdot}9027 * 10^9$
Out of sample				
$\frac{n}{N}$	0.0144	0.0187	0.0163	0.0152
b	$3{\cdot}4391 * 10^9$	$5{\cdot}4928 * 10^9$	$1{\cdot}3480 * 10^{10}$	$5{\cdot}0534 * 10^9$

significant part of the bidders' decision-making process and, hence, a theoretical model or an empirical analysis that ignores the entry effect may lead to misleading policy recommendations.

We have also used the structural estimates to simulate the winner's payoff defined as the winning bid less the sum of the private cost and the entry cost, as well as the winner's information rent defined as the ratio between the winner's payoff and the winning bid. It turns out that the average winner's payoff is $40,020 and the average information rent is 0·26.

6.2. *Density estimate for the unobserved heterogeneity*

In the Bayesian framework, an informative way to draw implications from the unknown densities of the error term $e_{new,\ell}$ is to study its predictive distribution since it summarizes the information on the data. Parallel to equation (C.2) in Appendix C, conditional on $(\theta_1, ..., \theta_L)$, for the new unit $\ell = L + 1$, we have

$$\theta_{L+1}|(\theta_1, ..., \theta_L) \sim \frac{\tau}{\tau + L} P_0 + \frac{1}{\tau + L} \sum_{j=1}^{m_c} n_j 1(\xi_j),$$

where n_j is the number of θ_i's taking the value ξ_j. Thus, the distribution of $e_{new,\ell+1}$ conditional on the data can be rewritten as

$$\begin{aligned} q(e_{new,L+1}|\theta_1, ..., \theta_L) &= \frac{\tau}{\tau + L} q_t(e_{new,L+1}|\mu_0, (1 + \tau_0)R_0/n_0, n_0) \\ &\quad + \frac{1}{\tau + L} \sum_{j=1}^{m_c} n_j \phi(e_{new,L+1}|\xi_j). \end{aligned}$$

Thus the predictive distribution for the error term $e_{new,L+1}$ can be obtained as

$$q(e_{new,L+1}|data) = \int q(e_{new,L+1}|\theta_1, ..., \theta_L)\pi(\theta_1, ..., \theta_L|data) d(\theta_1, ..., \theta_L).$$

Since the Gibbs sampler provides draws for θ_ℓ's, we can use the Monte Carlo method to integrate out θ_ℓ's to estimate $q(e_{new,L+1}|data)$ as

$$\widehat{q}(e_{new,L+1}|data) = \frac{1}{M} \sum_{i=1}^{M} q(e_{new,L+1}|\theta_1^{(i)}, ..., \theta_L^{(i)}),$$

where $(\theta_1^{(i)}, ..., \theta_L^{(i)})$ is a simulated sample of $(\theta_1, ..., \theta_L)$.

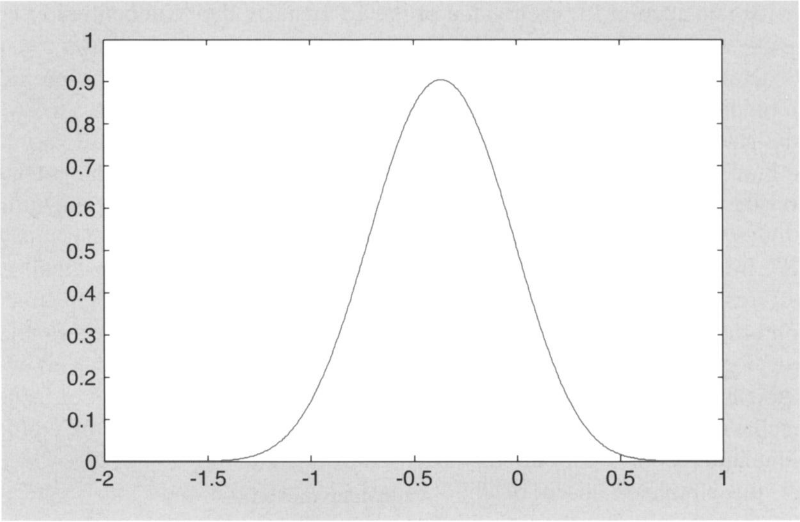

FIGURE 4
Estimated predictive density for e_{new}

Figure 4 plots the estimated predictive density for the unobserved term e_{new}. This indicates that there exists substantial unobserved heterogeneity in both the private value and entry cost distributions. The mode of the density is around $-0{\cdot}3$. Furthermore, the estimated density is asymmetric and slightly left skewed. This is evidence of the non-normality of the unobserved heterogeneity, which demonstrates the need for the non-parametric modelling of this density.

7. COUNTERFACTUAL ANALYSIS

7.1. *Quantifying the "competition effect" and the "entry effect" of the number of potential bidders N on b*

One interesting theoretical relationship we obtain from Section 3 is that in our models, the relationship between the number of potential bidders, N, and the individual bid, b, may not be monotone decreasing. This is due to the two effects of N on b. One is the "competition effect" and the other is the "entry effect". As given in Proposition 2, the "competition effect" is always negative and the "entry effect" is always positive. As a result, as the number of potential bidders increases, the equilibrium bid is increasing as well, provided that the positive "entry effect" dominates the negative "competition effect". If this is the case, policies that encourage more potential bidders might actually increase the equilibrium bid. One advantage of the structural approach is that it enables us to use the structural estimates to conduct a counterfactual analysis to quantify the "competition effect" and the "entry effect" of N on b separately.

To quantify the "competition effect" and the "entry effect", we run a counterfactual experiment for a representative auction. We pick the 123rd auction in our dataset. This auction has 11 potential bidders, which is the mean value for the number of potential bidders in our dataset. Also, the engineer's estimate for this auction is \$170,060, which is very close to the mean value of the engineer's estimate \$165,348·90. We use the last 1000 iterations of our MCMC algorithm for this counterfactual experiment. At each iteration, with the simulated values of μ_{1123} and k_{123}, where the subscript 123 denotes the 123rd auction, we can recover a unique q^*_{123} for this auction for each value of N. To quantify the "competition effect," we set the private cost c at $\exp(\overline{\mu_{1123}})$, where $\overline{\mu_{1123}}$ is the posterior mean of the draws of μ_{1123} during the MCMC iterations across all the iterations and then calculate the equilibrium bid using the equilibrium bidding function. We vary the number of potential bidders from 3 to 26, which are the minimum and the maximum observed values in the data. Note that as we vary the number of potential bidders from N to $N+1$, the q^*_{123} is fixed. For each value of the potential bidders N, we report the change in the equilibrium bid from N to $N+1$ as the "competition effect".

On the other hand, quantifying the "entry effect" directly is difficult, but quantifying the "total effect" from N to b is relatively straightforward. After we get the "total effect", the "entry effect" can be easily computed as the difference between the "total effect" and the "competition effect". To calculate the "total effect", we use almost the same simulation setup as in quantifying the "competition effect". The only difference here is that as we vary the number of potential bidders from N to $N+1$, we obtain a new q^*_{123} instead of using the fixed q^*_{123}. For each value of the potential bidders N, we report the change in the equilibrium bid from this simulation as the "total effect".

We plot the simulated mean of the "competition effect", the "entry effect" and the "total effect" and their corresponding confidence bands from the counterfactual experiment in Figure 5. The x-axis denotes the number of potential bidders N and the y-axis represents the change in the equilibrium bid from increasing the number of potential bidders from N to $N+1$. Figure 5 reveals several interesting results. First, an obvious feature of the plot is that

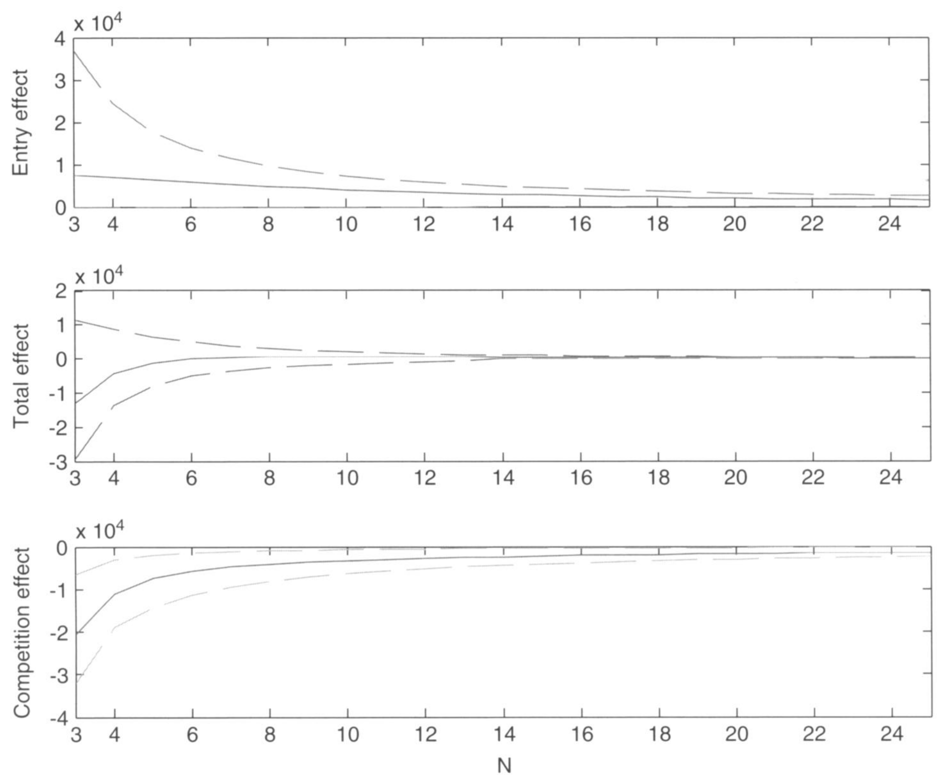

FIGURE 5

The "competition effect", "entry effect" and "total effect" from N on b

when the number of potential bidders increases, the "competition effect" is always negative. This is consistent with the theory since as the number of potential bidders increases and the equilibrium entry probability is fixed, bidders would anticipate more actual bidders and bid more aggressively. Second, the "entry effect" is always positive, which is also consistent with what our model implies. This shows that the entry behaviour in these auctions actually makes the bidders bid less aggressively, all else being equal. Third, the simulated mean of "total effect" from N on b is positive for $N \geq 7$. This demonstrates that in our simulation setup, on average, the positive "entry effect" weakly dominates the negative "competition effect" for a wide range of the number of potential bidders. In more detail, when $N = 7$, the simulated mean of the equilibrium bid for this representative auction is \$146,431·45. But the simulated mean of the equilibrium bid becomes \$149,704·62 (or 1·83% increase) when $N = 12$ and \$154,328·57 (or 5·39% increase) when $N = 26$. This finding is interesting since in the usual IPV auction model without entry, bidders will bid more aggressively when facing a larger number of potential bidders. In our case, however, when endogenous entry matters, the bids actually become less aggressive.

7.2. *Quantifying the "competition effect" and the "entry effect" of the number of potential bidders N on procurement costs*

Another interesting theoretical relationship we obtain from Section 3 is that in our models, the relationship between the number of potential bidders, N, and the procurement cost, W, may not be monotone decreasing. Again, this is due to the two effects of N on W. One is the "competition effect" and the other is the "entry effect". As given in Proposition 3, the "competition effect" is always negative and the "entry effect" is always positive. Thus, as the number of potential bidders increases, the government's procurement cost can be increasing as well, provided that the positive "entry effect" dominates the negative "competition effect". This means that policies that encourage more potential bidders might actually increase the government's procurement costs. We now use the structural estimates to conduct a counterfactual analysis to quantify the "competition effect" and the "entry effect" of N on the procurement costs separately.

To quantify the "competition effect" and the "entry effect" we again run a counterfactual experiment for the same representative auction. As in the previous subsection, we use the last 1000 iterations of our MCMC algorithm for this counterfactual experiment. At each iteration, with the simulated values of μ_{1123} and k_{123}, we can recover a unique q^*_{123} for this auction for each value of N. To quantify the "competition effect", we compute the expected winning bid. We vary the number of potential bidders from 3 to 26. Note that as we vary the number of potential bidders from N to $N+1$, the q^*_{123} is fixed. For each value of the potential bidders N, we report the change in the expected winning bid from N to $N+1$ as the "competition effect".

On the other hand, since quantifying the "entry effect" directly is difficult, we again first quantify the "total effect" from N to the procurement cost, which is relatively straightforward. After we get the "total effect", the "entry effect" can be easily computed as the difference between the "total effect" and the "competition effect". To calculate the "total effect", we use almost the same setup as in quantifying the "competition effect". The only difference here is that as we vary the number of potential bidders from N to $N+1$, we obtain a new q^*_{123} instead of using the fixed q^*_{123}. For each value of the number of potential bidders N, we report the change in the expected winning bid as the "total effect".

We plot the simulated mean of the "competition effect", the "entry effect" and the "total effect" and their corresponding confidence bands from the simulation in Figure 6. The x-axis

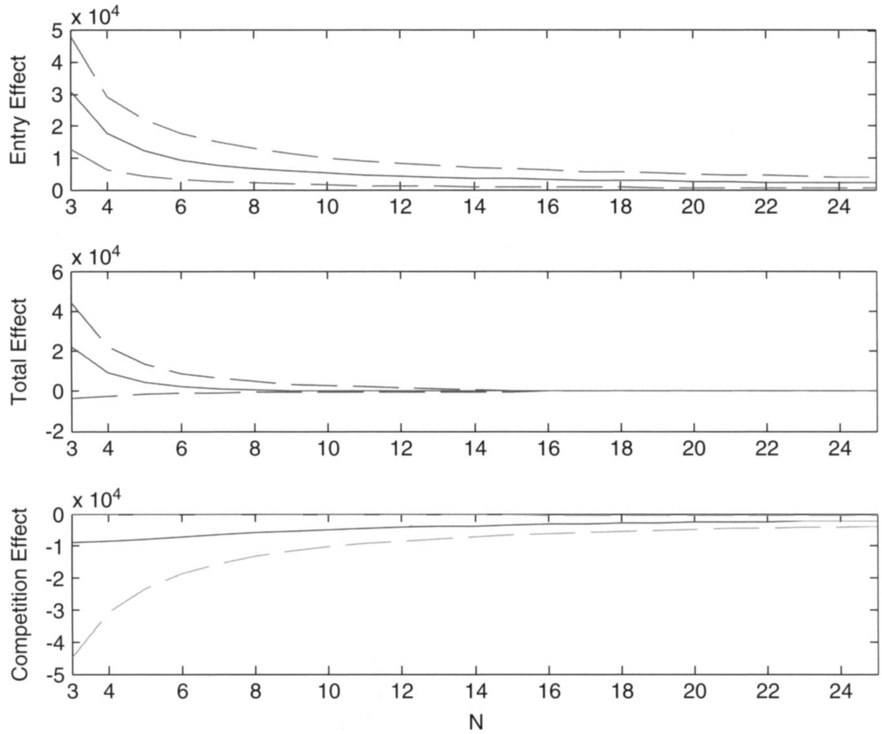

FIGURE 6

The "Competition Effect", "Entry Effect" and "Total Effect" from N on Procurement Cost

denotes the number of potential bidders N and the y-axis represents the change in the expected procurement cost from increasing the number of potential bidders from N to $N + 1$. Figure 6 reveals several interesting results. First, an obvious feature of the plot is that when the number of potential bidders increases, the "competition effect" is always negative and the "entry effect" is always positive. Second, the positive "entry effect" significantly dominates the negative "competition effect", leading to a positive "total effect" for a wide range of the number of potential bidders. This implies that the relationship between N and W is not monotone decreasing. We also find from the experiment that the expected winning bid hits its lowest point when $N = 3$, which is \$112,390·01 for this representative auction. But the expected winning bid becomes \$152,666·15 (or a 35·84% increase) when $N = 12$ and \$152,165·71 (or a 35·39% increase) when $N = 26$. This interesting finding indicates that with endogenous entry, the procurement cost can actually become greater as the number of potential bidders increases. Thus, in this kind of auction environment, policies that encourage more potential bidders may not be desirable.

7.3. *Quantifying the savings on procurement costs by reducing the entry cost*

From the estimation results, we find strong evidence that part of the bids is used to recover bidders' entry cost. Thus, it is interesting to quantify the savings in the government's procurement costs by reducing bidders' entry cost. This has important policy implications since if the savings are substantial, then the government can improve social welfare by designing mechanisms to reduce the entry cost.

To quantify the savings, we again run a counterfactual experiment for the same representative auction. We first set the entry cost k at $\exp(\overline{\mu_{2123}})$, where $\overline{\mu_{2123}}$ is the posterior mean of the draws of μ_{2123} during the MCMC iterations across all the iterations and u_{1123} set at similarly defined $\overline{\mu_{1123}}$. This allows us to recover a unique q^*_{123} for this auction and thus compute the expected winning bid. We then re-do the analysis by setting the entry cost k at 20, 50 and 80% of its original level, that is, at $0{\cdot}2 * \exp(\overline{\mu_{2123}})$, $0{\cdot}5 * \exp(\overline{\mu_{2123}})$ and $0{\cdot}8 * \exp(\overline{\mu_{2123}})$ respectively. This difference between the two settings can be regarded as the savings on procurement costs by reducing the entry cost. Our results show that by reducing the entry cost to half of its original level, the government can save approximately 28·18% of the procurement cost for this particular auction. The savings are 44·49% and 7·19% of the procurement cost if the entry cost is reduced to be 20% and 80% of its original level, respectively.

8. CONCLUSION

In this paper, we propose three models for procurement auctions with entry within the IPV paradigm. Our models yield several interesting implications, which are supported by the data we analyse. Most importantly, for the first time, we show that even within the IPV paradigm, as the number of potential bidders increases, bidders' equilibrium bidding behaviour can become less aggressive and the procurement cost can rise. Thus, increasing potential competition may not necessarily benefit the auctioneer. Whether this is the case is an empirical question that can be answered through a rigorous structural analysis, as demonstrated from our paper.

In order to answer this question among others that may be of interest to the auctioneer or/and policy makers, we develop a fully structural approach for entry and bidding based on the game-theoretic models of entry and bidding proposed in this paper. To circumvent the complexity associated with our structural models for entry and bidding and the need for controlling for unobserved auction heterogeneity, we propose a new semi-parametric Bayesian estimation algorithm to estimate the structural elements of the models. In particular, our structural approach allows us to recover the entry cost, which turns out to be about 8% of the winning bid. While this figure is comparable to the one found in Bajari, Hong and Ryan (2007), the nature of this relatively large entry cost is an open issue and an important one for the literature on auctions with entry that merits further investigation. With our structural approach, we are able to control for the effect of unobserved auction heterogeneity, and quantify the "entry effect" and the "competition effect". Therefore, our paper contributes to the literature by providing a unified framework to study procurement auctions with entry and unobserved auction heterogeneity, and sheds light on empirical analyses of other auctions.

Another related issue of importance is possible collusion, which the paper does not attempt to address. Throughout, we have maintained a key assumption that potential bidders do not collude. Therefore, all the results and conclusions can be sensitive to a change in this assumption. In particular, the estimated relatively large entry cost could be driven by the possibility that some potential bidders collude by refraining from bidding in certain auctions making it look like there are high entry costs.[28] While there has been evidence that cartels in procurement auctions exist (*e.g.* Porter and Zona, 1993), empirical studies of cartels and collusion in procurement auctions have been limited. The structural approach developed in this paper could be extended to study collusive behaviours among potential bidders taking entry into account. This is left for future research.

28. We are grateful to Editor Andrea Prat for this point.

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